

12

Lubrication and Journal Bearings

Chapter Outline

12-1	Types of Lubrication	610
12-2	Viscosity	611
12-3	Petroff's Equation	613
12-4	Stable Lubrication	615
12-5	Thick-Film Lubrication	616
12-6	Hydrodynamic Theory	617
12-7	Design Considerations	621
12-8	The Relations of the Variables	623
12-9	Steady-State Conditions in Self-Contained Bearings	637
12-10	Clearance	640
12-11	Pressure-Fed Bearings	642
12-12	Loads and Materials	648
12-13	Bearing Types	650
12-14	Thrust Bearings	651
12-15	Boundary-Lubricated Bearings	652

The object of lubrication is to reduce friction, wear, and heating of machine parts that move relative to each other. A lubricant is any substance that, when inserted between the moving surfaces, accomplishes these purposes. In a sleeve bearing, a shaft, or *journal*, rotates or oscillates within a sleeve, or *bushing*, and the relative motion is sliding. In an antifriction bearing, the main relative motion is rolling. A follower may either roll or slide on the cam. Gear teeth mate with each other by a combination of rolling and sliding. Pistons slide within their cylinders. All these applications require lubrication to reduce friction, wear, and heating.

The field of application for journal bearings is immense. The crankshaft and connecting-rod bearings of an automotive engine must operate for thousands of miles at high temperatures and under varying load conditions. The journal bearings used in the steam turbines of power-generating stations are said to have reliabilities approaching 100 percent. At the other extreme there are thousands of applications in which the loads are light and the service relatively unimportant; a simple, easily installed bearing is required, using little or no lubrication. In such cases an antifriction bearing might be a poor answer because of the cost, the elaborate enclosures, the close tolerances, the radial space required, the high speeds, or the increased inertial effects. Instead, a nylon bearing requiring no lubrication, a powder-metallurgy bearing with the lubrication “built in,” or a bronze bearing with ring oiling, wick feeding, or solid-lubricant film or grease lubrication might be a very satisfactory solution. Recent metallurgy developments in bearing materials, combined with increased knowledge of the lubrication process, now make it possible to design journal bearings with satisfactory lives and very good reliabilities.

Much of the material we have studied thus far in this book has been based on fundamental engineering studies, such as statics, dynamics, the mechanics of solids, metal processing, mathematics, and metallurgy. In the study of lubrication and journal bearings, additional fundamental studies, such as chemistry, fluid mechanics, thermodynamics, and heat transfer, must be utilized in developing the material. While we shall not utilize all of them in the material to be included here, you can now begin to appreciate better how the study of mechanical engineering design is really an integration of most of your previous studies and a directing of this total background toward the resolution of a single objective.

12-1 Types of Lubrication

Five distinct forms of lubrication may be identified:

- 1 Hydrodynamic
- 2 Hydrostatic
- 3 Elastohydrodynamic
- 4 Boundary
- 5 Solid film

Hydrodynamic lubrication means that the load-carrying surfaces of the bearing are separated by a relatively thick film of lubricant, so as to prevent metal-to-metal contact, and that the stability thus obtained can be explained by the laws of fluid mechanics. Hydrodynamic lubrication does not depend upon the introduction of the lubricant under pressure, though that may occur; but it does require the existence of an adequate supply at all times. The film pressure is created by the moving surface itself pulling the lubricant into a wedge-shaped zone at a velocity sufficiently high to create the pressure necessary to separate the surfaces against the load on the bearing. Hydrodynamic lubrication is also called *full-film*, or *fluid, lubrication*.

Hydrostatic lubrication is obtained by introducing the lubricant, which is sometimes air or water, into the load-bearing area at a pressure high enough to separate the surfaces with a relatively thick film of lubricant. So, unlike hydrodynamic lubrication, this kind of lubrication does not require motion of one surface relative to another. We shall not deal with hydrostatic lubrication in this book, but the subject should be considered in designing bearings where the velocities are small or zero and where the frictional resistance is to be an absolute minimum.

Elastohydrodynamic lubrication is the phenomenon that occurs when a lubricant is introduced between surfaces that are in rolling contact, such as mating gears or rolling bearings. The mathematical explanation requires the Hertzian theory of contact stress and fluid mechanics.

Insufficient surface area, a drop in the velocity of the moving surface, a lessening in the quantity of lubricant delivered to a bearing, an increase in the bearing load, or an increase in lubricant temperature resulting in a decrease in viscosity—any one of these—may prevent the buildup of a film thick enough for full-film lubrication. When this happens, the highest asperities may be separated by lubricant films only several molecular dimensions in thickness. This is called *boundary lubrication*. The change from hydrodynamic to boundary lubrication is not at all a sudden or abrupt one. It is probable that a mixed hydrodynamic- and boundary-type lubrication occurs first, and as the surfaces move closer together, the boundary-type lubrication becomes predominant. The viscosity of the lubricant is not of as much importance with boundary lubrication as is the chemical composition.

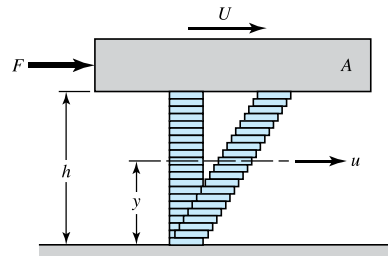
When bearings must be operated at extreme temperatures, a *solid-film lubricant* such as graphite or molybdenum disulfide must be used because the ordinary mineral oils are not satisfactory. Much research is currently being carried out in an effort to find composite bearing materials with low wear rates as well as small frictional coefficients.

12-2 Viscosity

In Fig. 12-1 let plate *A* be moving with a velocity *U* on a film of lubricant of thickness *h*. We imagine the film as composed of a series of horizontal layers and the force *F* causing these layers to deform or slide on one another just like a deck of cards. The layers in contact with the moving plate are assumed to have a velocity *U*; those in contact with the stationary surface are assumed to have a zero velocity. Intermediate layers have velocities that depend upon their distances *y* from the stationary surface. Newton's viscous effect states that the shear stress in the fluid is proportional to the rate of change of velocity with respect to *y*. Thus

$$\tau = \frac{F}{A} = \mu \frac{du}{dy} \quad (12-1)$$

Figure 12-1



where μ is the constant of proportionality and defines *absolute viscosity*, also called *dynamic viscosity*. The derivative du/dy is the rate of change of velocity with distance and may be called the rate of shear, or the velocity gradient. The viscosity μ is thus a measure of the internal frictional resistance of the fluid. For most lubricating fluids, the rate of shear is constant, and $du/dy = U/h$. Thus, from Eq. (12-1),

$$\tau = \frac{F}{A} = \mu \frac{U}{h} \quad (12-2)$$

Fluids exhibiting this characteristic are said to be *Newtonian fluids*. The unit of viscosity in the ips system is seen to be the pound-force-second per square inch; this is the same as stress or pressure multiplied by time. The ips unit is called the *reyn*, in honor of Sir Osborne Reynolds.

The absolute viscosity is measured by the pascal-second ($\text{Pa} \cdot \text{s}$) in SI; this is the same as a Newton-second per square meter. The conversion from ips units to SI is the same as for stress. For example, multiply the absolute viscosity in reyns by 6890 to convert to units of $\text{Pa} \cdot \text{s}$.

The American Society of Mechanical Engineers (ASME) has published a list of cgs units that are not to be used in ASME documents.¹ This list results from a recommendation by the International Committee of Weights and Measures (CIPM) that the use of cgs units with special names be discouraged. Included in this list is a unit of force called the *dyne* (dyn), a unit of dynamic viscosity called the *poise* (P), and a unit of kinematic viscosity called the *stoke* (St). All of these units have been, and still are, used extensively in lubrication studies.

The poise is the cgs unit of dynamic or absolute viscosity, and its unit is the dyne-second per square centimeter ($\text{dyn} \cdot \text{s}/\text{cm}^2$). It has been customary to use the centipoise (cP) in analysis, because its value is more convenient. When the viscosity is expressed in centipoises, it is designated by Z . The conversion from cgs units to SI and ips units is as follows:

$$\mu(\text{Pa} \cdot \text{s}) = (10)^{-3} Z (\text{cP})$$

$$\mu(\text{reyn}) = \frac{Z (\text{cP})}{6.89(10)^6}$$

$$\mu(\text{mPa} \cdot \text{s}) = 6.89 \mu' (\mu\text{reyn})$$

In using ips units, the microreyn (μreyn) is often more convenient. The symbol μ' will be used to designate viscosity in μreyn such that $\mu = \mu'/(10^6)$.

The ASTM standard method for determining viscosity uses an instrument called the Saybolt Universal Viscosimeter. The method consists of measuring the time in seconds for 60 mL of lubricant at a specified temperature to run through a tube 17.6 mm in diameter and 12.25 mm long. The result is called the *kinematic viscosity*, and in the past the unit of the square centimeter per second has been used. One square centimeter per second is defined as a *stoke*. By the use of the *Hagen-Poiseuille law*, the kinematic viscosity based upon seconds Saybolt, also called *Saybolt Universal viscosity* (SUV) in seconds, is

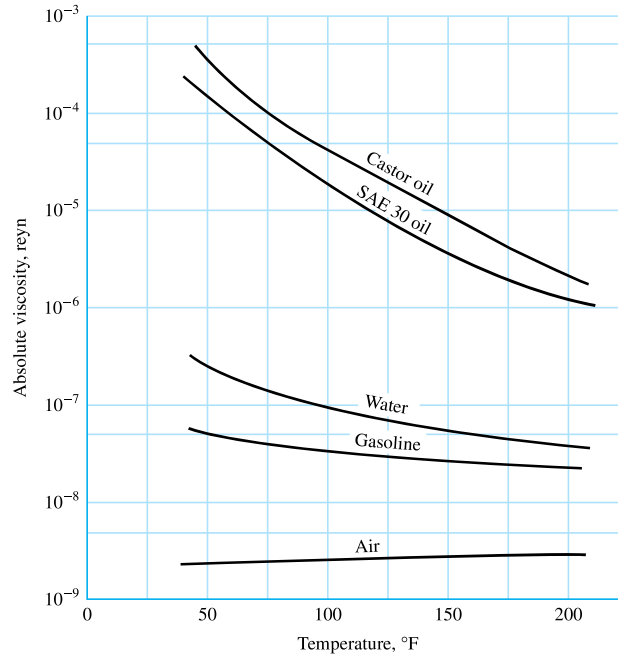
$$Z_k = \left(0.22t - \frac{180}{t} \right) \quad (12-3)$$

where Z_k is in centistokes (cSt) and t is the number of seconds Saybolt.

¹ASME *Orientation and Guide for Use of Metric Units*, 2nd ed., American Society of Mechanical Engineers, 1972, p. 13.

Figure 12-2

A comparison of the viscosities of various fluids.



In SI, the kinematic viscosity ν has the unit of the square meter per second (m^2/s), and the conversion is

$$\nu(\text{m}^2/\text{s}) = 10^{-6} Z_k (\text{cSt})$$

Thus, Eq. (12-3) becomes

$$\nu = \left(0.22t - \frac{180}{t} \right) (10^{-6}) \quad (12-4)$$

To convert to dynamic viscosity, we multiply ν by the density in SI units. Designating the density as ρ with the unit of the kilogram per cubic meter, we have

$$\mu = \rho \left(0.22t - \frac{180}{t} \right) (10^{-6}) \quad (12-5)$$

where μ is in pascal-seconds.

Figure 12-2 shows the absolute viscosity in the ips system of a number of fluids often used for lubrication purposes and their variation with temperature.

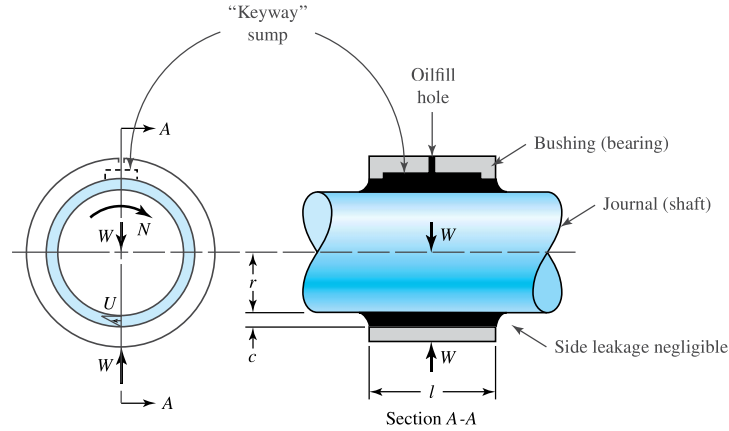
12-3 Petroff's Equation

The phenomenon of bearing friction was first explained by Petroff on the assumption that the shaft is concentric with its bushing. Though we shall seldom make use of Petroff's method of analysis in the material to follow, it is important because it defines groups of dimensionless parameters and because the coefficient of friction predicted by this law turns out to be quite good even when the shaft is not concentric.

Let us now consider a vertical shaft rotating in a guide bearing. It is assumed that the bearing carries a very small load, that the clearance space is completely filled with oil, and that leakage is negligible (Fig. 12-3). We denote the radius of the shaft

Figure 12-3

Petroff's lightly loaded journal bearing consisting of a shaft journal and a bushing with an axial-groove internal lubricant reservoir. The linear velocity gradient is shown in the end view. The clearance c is several thousandths of an inch and is grossly exaggerated for presentation purposes.



by r , the radial clearance by c , and the length of the bearing by l , all dimensions being in inches. If the shaft rotates at N rev/s, then its surface velocity is $U = 2\pi rN$ in/s. Since the shearing stress in the lubricant is equal to the velocity gradient times the viscosity, from Eq. (12-2) we have

$$\tau = \mu \frac{U}{h} = \frac{2\pi r \mu N}{c} \quad (a)$$

where the radial clearance c has been substituted for the distance h . The force required to shear the film is the stress times the area. The torque is the force times the lever arm r . Thus

$$T = (\tau A)(r) = \left(\frac{2\pi r \mu N}{c} \right) (2\pi r l)(r) = \frac{4\pi^2 r^3 l \mu N}{c} \quad (b)$$

If we now designate a small force on the bearing by W , in pounds-force, then the pressure P , in pounds-force per square inch of projected area, is $P = W/2rl$. The frictional force is fW , where f is the coefficient of friction, and so the frictional torque is

$$T = fWr = (f)(2rlP)(r) = 2r^2 f l P \quad (c)$$

Substituting the value of the torque from Eq. (c) in Eq. (b) and solving for the coefficient of friction, we find

$$f = 2\pi^2 \frac{\mu N}{P} \frac{r}{c} \quad (12-6)$$

Equation (12-6) is called *Petroff's equation* and was first published in 1883. The two quantities $\mu N/P$ and r/c are very important parameters in lubrication. Substitution of the appropriate dimensions in each parameter will show that they are dimensionless.

The *bearing characteristic number*, or the *Sommerfeld number*, is defined by the equation

$$S = \left(\frac{r}{c} \right)^2 \frac{\mu N}{P} \quad (12-7)$$

The Sommerfeld number is very important in lubrication analysis because it contains many of the parameters that are specified by the designer. Note that it is also dimensionless. The quantity r/c is called the *radial clearance ratio*. If we multiply both

sides of Eq. (12-6) by this ratio, we obtain the interesting relation

$$f \frac{r}{c} = 2\pi^2 \frac{\mu N}{P} \left(\frac{r}{c} \right)^2 = 2\pi^2 S \quad (12-8)$$

12-4 Stable Lubrication

The difference between boundary and hydrodynamic lubrication can be explained by reference to Fig. 12-4. This plot of the change in the coefficient of friction versus the bearing characteristic $\mu N/P$ was obtained by the McKee brothers in an actual test of friction.² The plot is important because it defines stability of lubrication and helps us to understand hydrodynamic and boundary, or thin-film, lubrication.

Recall Petroff's bearing model in the form of Eq. (12-6) predicts that f is proportional to $\mu N/P$, that is, a straight line from the origin in the first quadrant. On the coordinates of Fig. 12-4 the locus to the right of point C is an example. Petroff's model presumes thick-film lubrication, that is, no metal-to-metal contact, the surfaces being completely separated by a lubricant film.

The McKee abscissa was ZN/P (centipoise \times rev/min/psi) and the value of abscissa B in Fig. 12-4 was 30. The corresponding $\mu N/P$ (reyn \times rev/s/psi) is $0.33(10^{-6})$. Designers keep $\mu N/P \geq 1.7(10^{-6})$, which corresponds to $ZN/P \geq 150$. A design constraint to keep thick-film lubrication is to be sure that

$$\frac{\mu N}{P} \geq 1.7(10^{-6}) \quad (a)$$

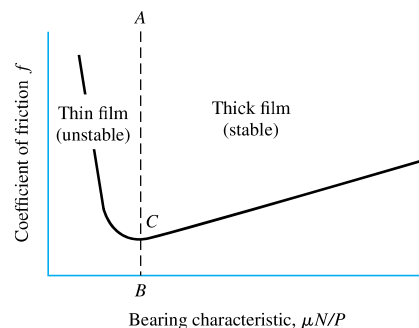
Suppose we are operating to the right of line BA and something happens, say, an increase in lubricant temperature. This results in a lower viscosity and hence a smaller value of $\mu N/P$. The coefficient of friction decreases, not as much heat is generated in shearing the lubricant, and consequently the lubricant temperature drops. Thus the region to the right of line BA defines *stable lubrication* because variations are self-correcting.

To the left of line BA , a decrease in viscosity would increase the friction. A temperature rise would ensue, and the viscosity would be reduced still more. The result would be compounded. Thus the region to the left of line BA represents *unstable lubrication*.

It is also helpful to see that a small viscosity, and hence a small $\mu N/P$, means that the lubricant film is very thin and that there will be a greater possibility of some metal-to-metal contact, and hence of more friction. Thus, point C represents what is probably the beginning of metal-to-metal contact as $\mu N/P$ becomes smaller.

Figure 12-4

The variation of the coefficient of friction f with $\mu N/P$.



²S. A. McKee and T. R. McKee, "Journal Bearing Friction in the Region of Thin Film Lubrication," *SAE J.*, vol. 31, 1932, pp. (T)371-377.

12-5 Thick-Film Lubrication

Let us now examine the formation of a lubricant film in a journal bearing. Figure 12-5a shows a journal that is just beginning to rotate in a clockwise direction. Under starting conditions, the bearing will be dry, or at least partly dry, and hence the journal will climb or roll up the right side of the bearing as shown in Fig. 12-5a.

Now suppose a lubricant is introduced into the top of the bearing as shown in Fig. 12-5b. The action of the rotating journal is to pump the lubricant around the bearing in a clockwise direction. The lubricant is pumped into a wedge-shaped space and forces the journal over to the other side. A *minimum film thickness* h_0 occurs, not at the bottom of the journal, but displaced clockwise from the bottom as in Fig. 12-5b. This is explained by the fact that a film pressure in the converging half of the film reaches a maximum somewhere to the left of the bearing center.

Figure 12-5 shows how to decide whether the journal, under hydrodynamic lubrication, is eccentrically located on the right or on the left side of the bearing. Visualize the journal beginning to rotate. Find the side of the bearing upon which the journal tends to roll. Then, if the lubrication is hydrodynamic, mentally place the journal on the opposite side.

The nomenclature of a journal bearing is shown in Fig. 12-6. The dimension c is the *radial clearance* and is the difference in the radii of the bushing and journal. In Fig. 12-6 the center of the journal is at O and the center of the bearing at O' . The

Figure 12-5

Formation of a film.

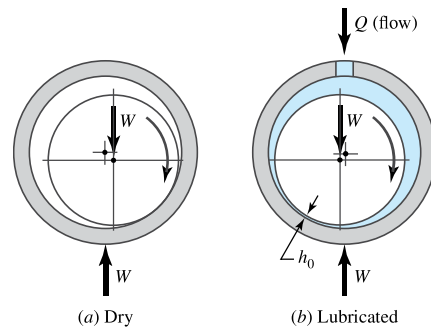
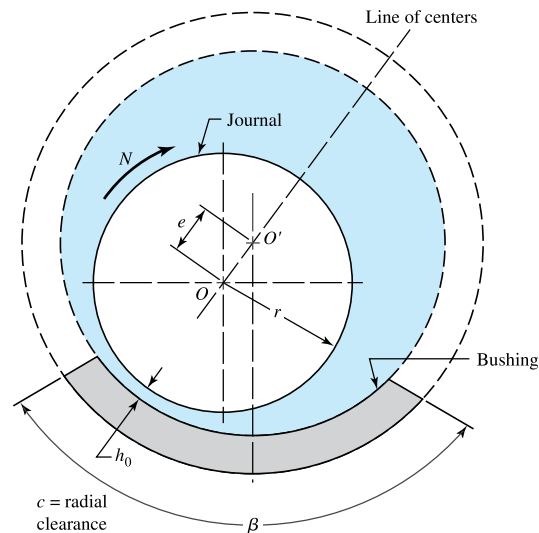


Figure 12-6

Nomenclature of a partial journal bearing.



distance between these centers is the *eccentricity* and is denoted by e . The *minimum film thickness* is designated by h_0 , and it occurs at the line of centers. The film thickness at any other point is designated by h . We also define an *eccentricity ratio* ϵ as

$$\epsilon = \frac{e}{c}$$

The bearing shown in the figure is known as a *partial bearing*. If the radius of the bushing is the same as the radius of the journal, it is known as a *fitted bearing*. If the bushing encloses the journal, as indicated by the dashed lines, it becomes a *full bearing*. The angle β describes the angular length of a partial bearing. For example, a 120° partial bearing has the angle β equal to 120° .

12-6 Hydrodynamic Theory

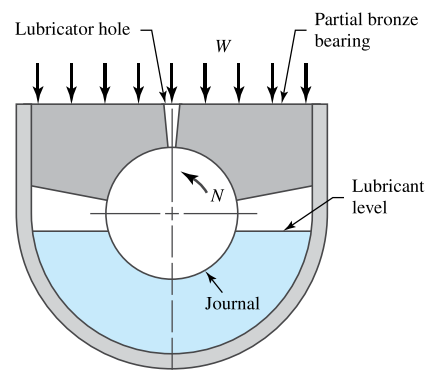
The present theory of hydrodynamic lubrication originated in the laboratory of Beauchamp Tower in the early 1880s in England. Tower had been employed to study the friction in railroad journal bearings and learn the best methods of lubricating them. It was an accident or error, during the course of this investigation, that prompted Tower to look at the problem in more detail and that resulted in a discovery that eventually led to the development of the theory.

Figure 12-7 is a schematic drawing of the journal bearing that Tower investigated. It is a partial bearing, having a diameter of 4 in, a length of 6 in, and a bearing arc of 157° , and having bath-type lubrication, as shown. The coefficients of friction obtained by Tower in his investigations on this bearing were quite low, which is now not surprising. After testing this bearing, Tower later drilled a $\frac{1}{2}$ -in-diameter lubricator hole through the top. But when the apparatus was set in motion, oil flowed out of this hole. In an effort to prevent this, a cork stopper was used, but this popped out, and so it was necessary to drive a wooden plug into the hole. When the wooden plug was pushed out too, Tower, at this point, undoubtedly realized that he was on the verge of discovery. A pressure gauge connected to the hole indicated a pressure of more than twice the unit bearing load. Finally, he investigated the bearing film pressures in detail throughout the bearing width and length and reported a distribution similar to that of Fig. 12-8.³

The results obtained by Tower had such regularity that Osborne Reynolds concluded that there must be a definite equation relating the friction, the pressure, and

Figure 12-7

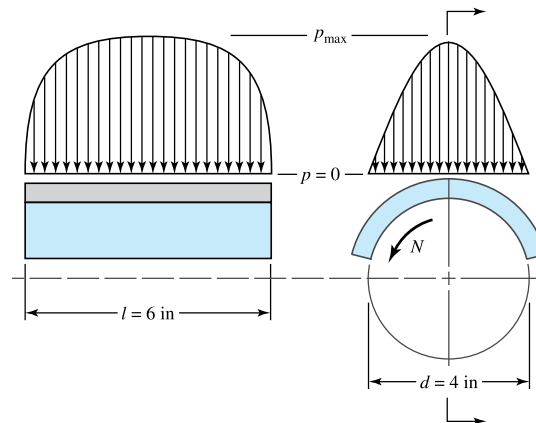
Schematic representation of the partial bearing used by Tower.



³Beauchamp Tower, "First Report on Friction Experiments," *Proc. Inst. Mech. Eng.*, November 1883, pp. 632–666; "Second Report," *ibid.*, 1885, pp. 58–70; "Third Report," *ibid.*, 1888, pp. 173–205; "Fourth Report," *ibid.*, 1891, pp. 111–140.

Figure 12-8

Approximate pressure-distribution curves obtained by Tower.



the velocity. The present mathematical theory of lubrication is based upon Reynolds' work following the experiment by Tower.⁴ The original differential equation, developed by Reynolds, was used by him to explain Tower's results. The solution is a challenging problem that has interested many investigators ever since then, and it is still the starting point for lubrication studies.

Reynolds pictured the lubricant as adhering to both surfaces and being pulled by the moving surface into a narrowing, wedge-shaped space so as to create a fluid or film pressure of sufficient intensity to support the bearing load. One of the important simplifying assumptions resulted from Reynolds' realization that the fluid films were so thin in comparison with the bearing radius that the curvature could be neglected. This enabled him to replace the curved partial bearing with a flat bearing, called a *plane slider bearing*. Other assumptions made were:

- 1 The lubricant obeys Newton's viscous effect, Eq. (12-1).
- 2 The forces due to the inertia of the lubricant are neglected.
- 3 The lubricant is assumed to be incompressible.
- 4 The viscosity is assumed to be constant throughout the film.
- 5 The pressure does not vary in the axial direction.

Figure 12-9a shows a journal rotating in the clockwise direction supported by a film of lubricant of variable thickness h on a partial bearing, which is fixed. We specify that the journal has a constant surface velocity U . Using Reynolds' assumption that curvature can be neglected, we fix a right-handed xyz reference system to the stationary bearing. We now make the following additional assumptions:

- 6 The bushing and journal extend infinitely in the z direction; this means there can be no lubricant flow in the z direction.
- 7 The film pressure is constant in the y direction. Thus the pressure depends only on the coordinate x .
- 8 The velocity of any particle of lubricant in the film depends only on the coordinates x and y .

We now select an element of lubricant in the film (Fig. 12-9a) of dimensions dx , dy , and dz , and compute the forces that act on the sides of this element. As shown in

⁴Osborne Reynolds, "Theory of Lubrication, Part I," *Phil. Trans. Roy. Soc. London*, 1886.

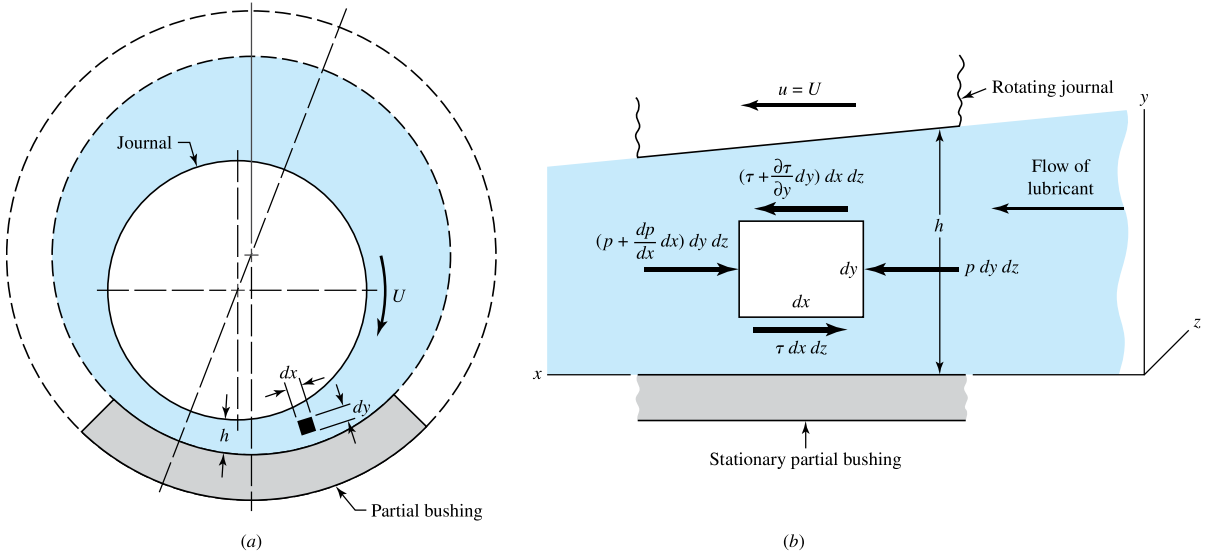


Figure 12-9

Fig. 12-9b, normal forces, due to the pressure, act upon the right and left sides of the element, and shear forces, due to the viscosity and to the velocity, act upon the top and bottom sides. Summing the forces in the x direction gives

$$\sum F_x = p \, dy \, dz - \left(p + \frac{dp}{dx} dx \right) dy \, dz - \tau \, dx \, dz + \left(\tau + \frac{\partial \tau}{\partial y} dy \right) dx \, dz = 0 \quad (a)$$

This reduces to

$$\frac{dp}{dx} = \frac{\partial \tau}{\partial y} \quad (b)$$

From Eq. (12-1), we have

$$\tau = \mu \frac{\partial u}{\partial y} \quad (c)$$

where the partial derivative is used because the velocity u depends upon both x and y . Substituting Eq. (c) in Eq. (b), we obtain

$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2} \quad (d)$$

Holding x constant, we now integrate this expression twice with respect to y . This gives

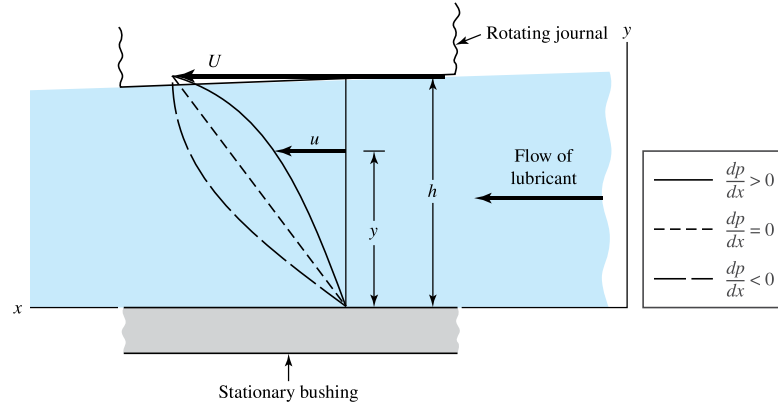
$$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{1}{\mu} \frac{dp}{dx} y + C_1 \\ u &= \frac{1}{2\mu} \frac{dp}{dx} y^2 + C_1 y + C_2 \end{aligned} \quad (e)$$

Note that the act of holding x constant means that C_1 and C_2 can be functions of x . We now assume that there is no slip between the lubricant and the boundary surfaces. This gives two sets of boundary conditions for evaluating C_1 and C_2 :

$$\begin{aligned} \text{At } y = 0, \quad u &= 0 \\ \text{At } y = h, \quad u &= U \end{aligned} \quad (f)$$

Figure 12-10

Velocity of the lubricant.



Notice, in the second condition, that h is a function of x . Substituting these conditions in Eq. (e) and solving for C_1 and C_2 gives

$$C_1 = \frac{U}{h} - \frac{h}{2\mu} \frac{dp}{dx} \quad C_2 = 0$$

or

$$u = \frac{1}{2\mu} \frac{dp}{dx} (y^2 - hy) + \frac{U}{h} y \quad (12-9)$$

This equation gives the velocity distribution of the lubricant in the film as a function of the coordinate y and the pressure gradient dp/dx . The equation shows that the velocity distribution across the film (from $y = 0$ to $y = h$) is obtained by superposing a parabolic distribution onto a linear distribution. Figure 12-10 shows the superposition of these distributions to obtain the velocity for particular values of x and dp/dx . In general, the parabolic term may be additive or subtractive to the linear term, depending upon the sign of the pressure gradient. When the pressure is maximum, $dp/dx = 0$ and the velocity is

$$u = \frac{U}{h} y \quad (g)$$

which is a linear relation.

We next define Q as the volume of lubricant flowing in the x direction per unit time. By using a width of unity in the z direction, the volume may be obtained by the expression

$$Q = \int_0^h u \, dy \quad (h)$$

Substituting the value of u from Eq. (12-9) and integrating gives

$$Q = \frac{Uh}{2} - \frac{h^3}{12\mu} \frac{dp}{dx} \quad (i)$$

The next step uses the assumption of an incompressible lubricant and states that the flow is the same for any cross section. Thus

$$\frac{dQ}{dx} = 0$$

From Eq. (i),

$$\frac{dQ}{dx} = \frac{U}{2} \frac{dh}{dx} - \frac{d}{dx} \left(\frac{h^3}{12\mu} \frac{dp}{dx} \right) = 0$$

or

$$\frac{d}{dx} \left(\frac{h^3}{\mu} \frac{dp}{dx} \right) = 6U \frac{dh}{dx} \quad (12-10)$$

which is the classical Reynolds equation for one-dimensional flow. It neglects side leakage, that is, flow in the z direction. A similar development is used when side leakage is not neglected. The resulting equation is

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 6U \frac{\partial h}{\partial x} \quad (12-11)$$

There is no general analytical solution to Eq. (12-11); approximate solutions have been obtained by using electrical analogies, mathematical summations, relaxation methods, and numerical and graphical methods. One of the important solutions is due to Sommerfeld⁵ and may be expressed in the form

$$\frac{r}{c} f = \phi \left[\left(\frac{r}{c} \right)^2 \frac{\mu N}{P} \right] \quad (12-12)$$

where ϕ indicates a functional relationship. Sommerfeld found the functions for half-bearings and full bearings by using the assumption of no side leakage.

12-7

Design Considerations

We may distinguish between two groups of variables in the design of sliding bearings. In the first group are those whose values either are given or are under the control of the designer. These are:

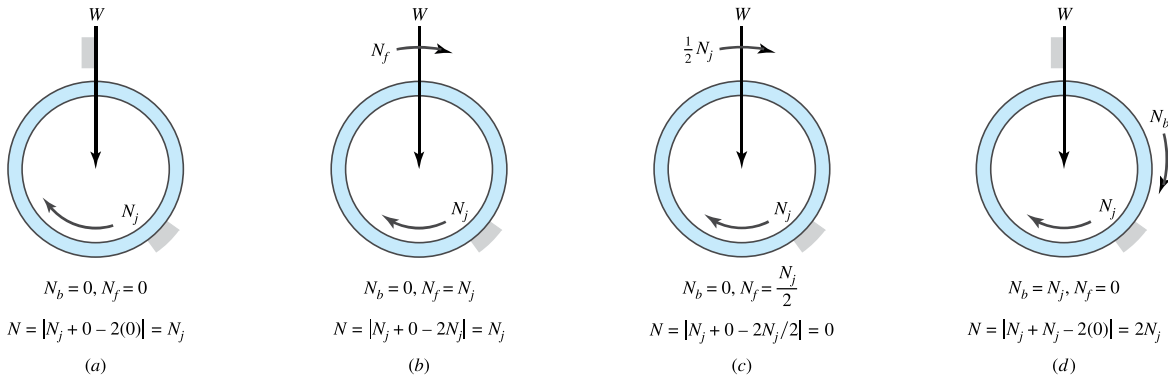
- 1 The viscosity μ
- 2 The load per unit of projected bearing area, P
- 3 The speed N
- 4 The bearing dimensions r , c , β , and l

Of these four variables, the designer usually has no control over the speed, because it is specified by the overall design of the machine. Sometimes the viscosity is specified in advance, as, for example, when the oil is stored in a sump and is used for lubricating and cooling a variety of bearings. The remaining variables, and sometimes the viscosity, may be controlled by the designer and are therefore the *decisions* the designer makes. In other words, when these four decisions are made, the design is complete.

In the second group are the dependent variables. The designer cannot control these except indirectly by changing one or more of the first group. These are:

- 1 The coefficient of friction f
- 2 The temperature rise ΔT
- 3 The volume flow rate of oil Q
- 4 The minimum film thickness h_0

⁵A. Sommerfeld, "Zur Hydrodynamischen Theorie der Schmiermittel-Reibung" ("On the Hydrodynamic Theory of Lubrication"), *Z. Math. Physik*, vol. 50, 1904, pp. 97-155.

**Figure 12-11**

How the significant speed varies. (a) Common bearing case. (b) Load vector moves at the same speed as the journal. (c) Load vector moves at half journal speed, no load can be carried. (d) Journal and bushing move at same speed, load vector stationary, capacity halved.

This group of variables tells us how well the bearing is performing, and hence we may regard them as *performance factors*. Certain limitations on their values must be imposed by the designer to ensure satisfactory performance. These limitations are specified by the characteristics of the bearing materials and of the lubricant. The fundamental problem in bearing design, therefore, is to define satisfactory limits for the second group of variables and then to decide upon values for the first group such that these limitations are not exceeded.

Significant Angular Speed

In the next section we will examine several important charts relating key variables to the Sommerfeld number. To this point we have assumed that only the journal rotates and it is the journal rotational speed that is used in the Sommerfeld number. It has been discovered that the angular speed N that is significant to hydrodynamic film bearing performance is⁶

$$N = |N_j + N_b - 2N_f| \quad (12-13)$$

where N_j = journal angular speed, rev/s

N_b = bearing angular speed, rev/s

N_f = load vector angular speed, rev/s

When determining the Sommerfeld number for a general bearing, use Eq. (12-13) when entering N . Figure 12-11 shows several situations for determining N .

Trumpler's Design Criteria for Journal Bearings

Because the bearing assembly creates the lubricant pressure to carry a load, it reacts to loading by changing its eccentricity, which reduces the minimum film thickness h_0 until the load is carried. What is the limit of smallness of h_0 ? Close examination reveals that the moving adjacent surfaces of the journal and bushing are not smooth but consist of a series of asperities that pass one another, separated by a lubricant film. In starting

⁶Paul Robert Trumpler, *Design of Film Bearings*, Macmillan, New York, 1966, pp. 103–119.

a bearing under load from rest there is metal-to-metal contact and surface asperities are broken off, free to move and circulate with the oil. Unless a filter is provided, this debris accumulates. Such particles have to be free to tumble at the section containing the minimum film thickness without snagging in a togglelike configuration, creating additional damage and debris. Trumpler, an accomplished bearing designer, provides a throat of at least $200\ \mu$ in to pass particles from ground surfaces.⁷ He also provides for the influence of size (tolerances tend to increase with size) by stipulating

$$h_0 \geq 0.0002 + 0.000\ 04d\ \text{in} \quad (a)$$

where d is the journal diameter in inches.

A lubricant is a mixture of hydrocarbons that reacts to increasing temperature by vaporizing the lighter components, leaving behind the heavier. This process (bearings have lots of time) slowly increases the viscosity of the remaining lubricant, which increases heat generation rate and elevates lubricant temperatures. This sets the stage for future failure. For light oils, Trumpler limits the maximum film temperature T_{\max} to

$$T_{\max} \leq 250^\circ\text{F} \quad (b)$$

Some oils can operate at slightly higher temperatures. Always check with the lubricant manufacturer.

A journal bearing often consists of a ground steel journal working against a softer, usually nonferrous, bushing. In starting under load there is metal-to-metal contact, abrasion, and the generation of wear particles, which, over time, can change the geometry of the bushing. The starting load divided by the projected area is limited to

$$\frac{W_{st}}{lD} \leq 300\ \text{psi} \quad (c)$$

If the load on a journal bearing is suddenly increased, the increase in film temperature in the annulus is immediate. Since ground vibration due to passing trucks, trains, and earth tremors is often present, Trumpler used a design factor of 2 or more on the running load, but not on the starting load of Eq. (c):

$$n_d \geq 2 \quad (d)$$

Many of Trumpler's designs are operating today, long after his consulting career was over; clearly they constitute good advice to the beginning designer.

12-8 The Relations of the Variables

Before proceeding to the problem of design, it is necessary to establish the relationships between the variables. Albert A. Raimondi and John Boyd, of Westinghouse Research Laboratories, used an iteration technique to solve Reynolds' equation on the digital computer.⁸ This is the first time such extensive data have been available for use by designers, and consequently we shall employ them in this book.⁹

⁷Op. cit., pp. 192–194.

⁸A. A. Raimondi and John Boyd, "A Solution for the Finite Journal Bearing and Its Application to Analysis and Design, Parts I, II, and III," *Trans. ASLE*, vol. 1, no. 1, in *Lubrication Science and Technology*, Pergamon, New York, 1958, pp. 159–209.

⁹See also the earlier companion paper, John Boyd and Albert A. Raimondi, "Applying Bearing Theory to the Analysis and Design of Journal Bearings, Parts I and II," *J. Appl. Mechanics*, vol. 73, 1951, pp. 298–316.

The Raimondi and Boyd papers were published in three parts and contain 45 detailed charts and 6 tables of numerical information. In all three parts, charts are used to define the variables for length-diameter (l/d) ratios of 1:4, 1:2, and 1 and for beta angles of 60 to 360°. Under certain conditions the solution to the Reynolds equation gives negative pressures in the diverging portion of the oil film. Since a lubricant cannot usually support a tensile stress, Part III of the Raimondi-Boyd papers assumes that the oil film is ruptured when the film pressure becomes zero. Part III also contains data for the infinitely long bearing; since it has no ends, this means that there is no side leakage. The charts appearing in this book are from Part III of the papers, and are for full journal bearings ($\beta = 360^\circ$) only. Space does not permit the inclusion of charts for partial bearings. This means that you must refer to the charts in the original papers when beta angles of less than 360° are desired. The notation is very nearly the same as in this book, and so no problems should arise.

Viscosity Charts (Figs. 12-12 to 12-14)

One of the most important assumptions made in the Raimondi-Boyd analysis is that *viscosity of the lubricant is constant as it passes through the bearing*. But since work is done on the lubricant during this flow, the temperature of the oil is higher when it leaves the loading zone than it was on entry. And the viscosity charts clearly indicate that the viscosity drops off significantly with a rise in temperature. Since the analysis is based on a constant viscosity, our problem now is to determine the value of viscosity to be used in the analysis.

Some of the lubricant that enters the bearing emerges as a side flow, which carries away some of the heat. The balance of the lubricant flows through the load-bearing zone and carries away the balance of the heat generated. In determining the viscosity to be used we shall employ a temperature that is the average of the inlet and outlet temperatures, or

$$T_{av} = T_1 + \frac{\Delta T}{2} \quad (12-14)$$

where T_1 is the inlet temperature and ΔT is the temperature rise of the lubricant from inlet to outlet. Of course, the viscosity used in the analysis must correspond to T_{av} .

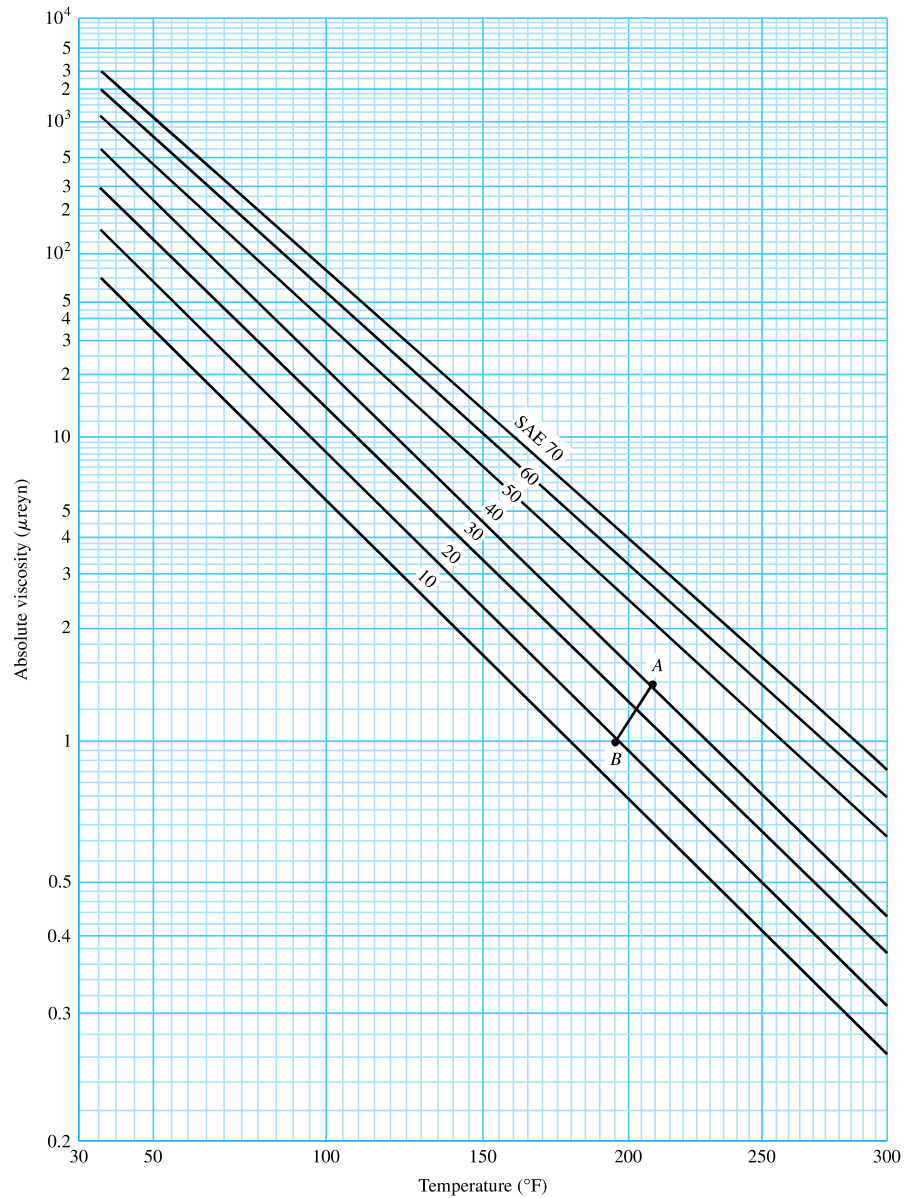
Viscosity varies considerably with temperature in a nonlinear fashion. The ordinates in Figs. 12-12 to 12-14 are not logarithmic, as the decades are of differing vertical length. These graphs represent the temperature versus viscosity functions for common grades of lubricating oils in both customary engineering and SI units. We have the temperature versus viscosity function only in graphical form, unless curve fits are developed. See Table 12-1.

One of the objectives of lubrication analysis is to determine the oil outlet temperature when the oil and its inlet temperature are specified. This is a trial-and-error type of problem. In an analysis, the temperature rise will first be estimated. This allows for the viscosity to be determined from the chart. With the value of the viscosity, the analysis is performed where the temperature rise is then computed. With this, a new estimate of the temperature rise is established. This process is continued until the estimated and computed temperatures agree.

To illustrate, suppose we have decided to use SAE 30 oil in an application in which the oil inlet temperature is $T_1 = 180^\circ\text{F}$. We begin by estimating that the

Figure 12-12

Viscosity–temperature chart
in U.S. customary units.
(Raimondi and Boyd.)



temperature rise will be $\Delta T = 30^\circ\text{F}$. Then, from Eq. (12-14),

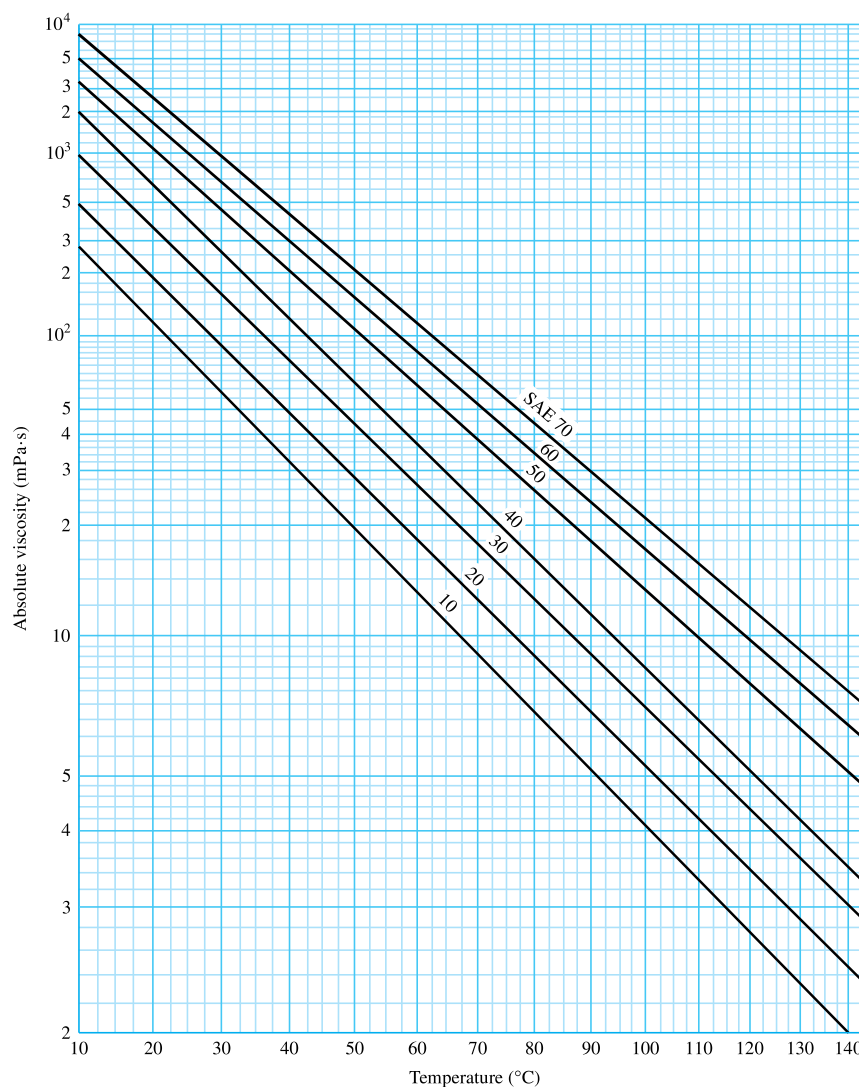
$$T_{\text{av}} = T_1 + \frac{\Delta T}{2} = 180 + \frac{30}{2} = 195^\circ\text{F}$$

From Fig. 12-12 we follow the SAE 30 line and find that $\mu = 1.40 \mu\text{reyn}$ at 195°F . So we use this viscosity (in an analysis to be explained in detail later) and find that the temperature rise is actually $\Delta T = 54^\circ\text{F}$. Thus Eq. (12-14) gives

$$T_{\text{av}} = 180 + \frac{54}{2} = 207^\circ\text{F}$$

Figure 12-13

Viscosity–temperature chart
in SI units. (Adapted from
Fig. 12-12.)



This corresponds to point *A* on Fig. 12-12, which is above the SAE 30 line and indicates that the viscosity used in the analysis was too high.

For a second guess, try $\mu = 1.00 \mu\text{reyn}$. Again we run through an analysis and this time find that $\Delta T = 30^\circ\text{F}$. This gives an average temperature of

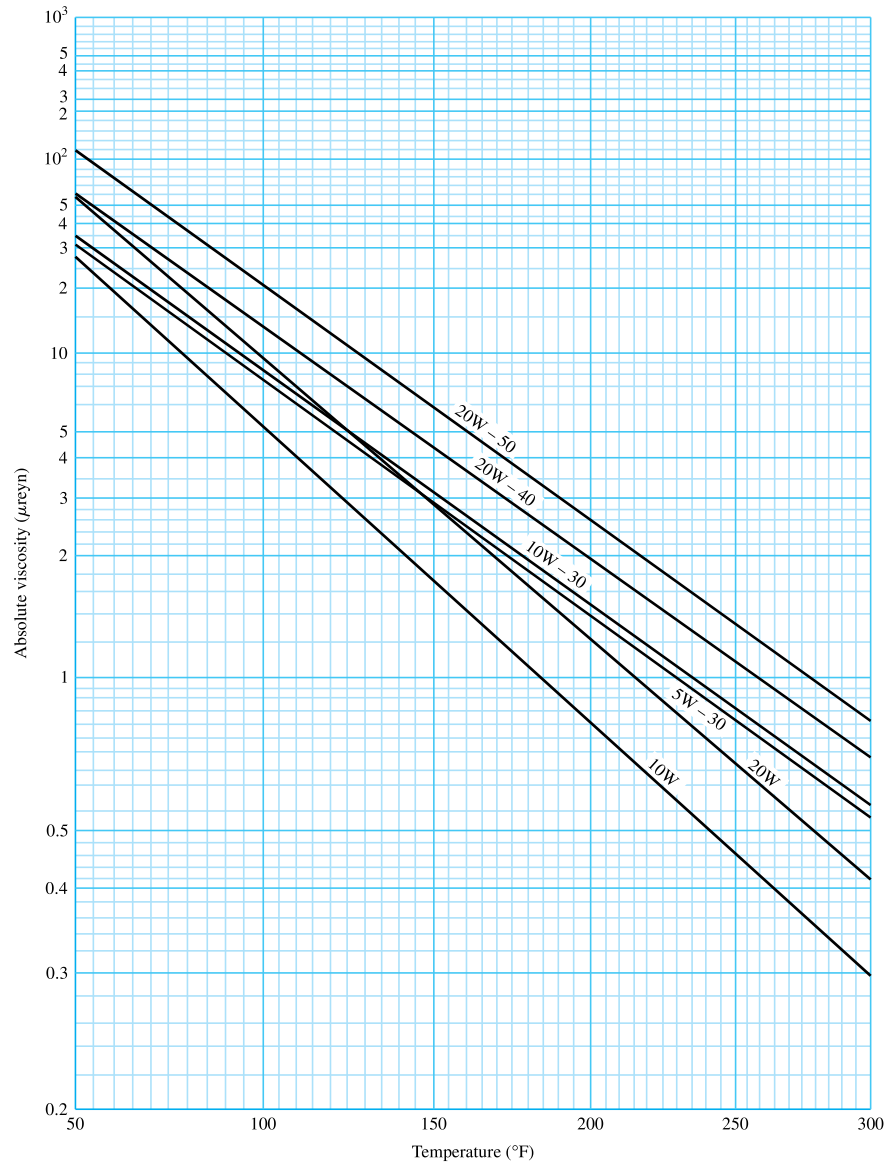
$$T_{\text{av}} = 180 + \frac{30}{2} = 195^\circ\text{F}$$

and locates point *B* on Fig. 12-12.

If points *A* and *B* are fairly close to each other and on opposite sides of the SAE 30 line, a straight line can be drawn between them with the intersection locating the correct values of viscosity and average temperature to be used in the analysis. For this illustration, we see from the viscosity chart that they are $T_{\text{av}} = 203^\circ\text{F}$ and $\mu = 1.20 \mu\text{reyn}$.

Figure 12-14

Chart for multiviscosity lubricants. This chart was derived from known viscosities at two points, 100 and 210°F, and the results are believed to be correct for other temperatures.

**Table 12-1**

Curve Fits* to Approximate the Viscosity versus Temperature Functions for SAE Grades 10 to 60

Source: A. S. Seireg and S. Dandage, "Empirical Design Procedure for the Thermodynamic Behavior of Journal Bearings," *J. Lubrication Technology*, vol. 104, April 1982, pp. 135-148.

Oil Grade, SAE	Viscosity μ_0 , reyn	Constant b , °F
10	$0.0158(10^{-6})$	1157.5
20	$0.0136(10^{-6})$	1271.6
30	$0.0141(10^{-6})$	1360.0
40	$0.0121(10^{-6})$	1474.4
50	$0.0170(10^{-6})$	1509.6
60	$0.0187(10^{-6})$	1564.0

* $\mu = \mu_0 \exp [b/(T + 95)]$, T in °F.

Figure 12-15

Polar diagram of the film-pressure distribution showing the notation used. (Raimondi and Boyd.)

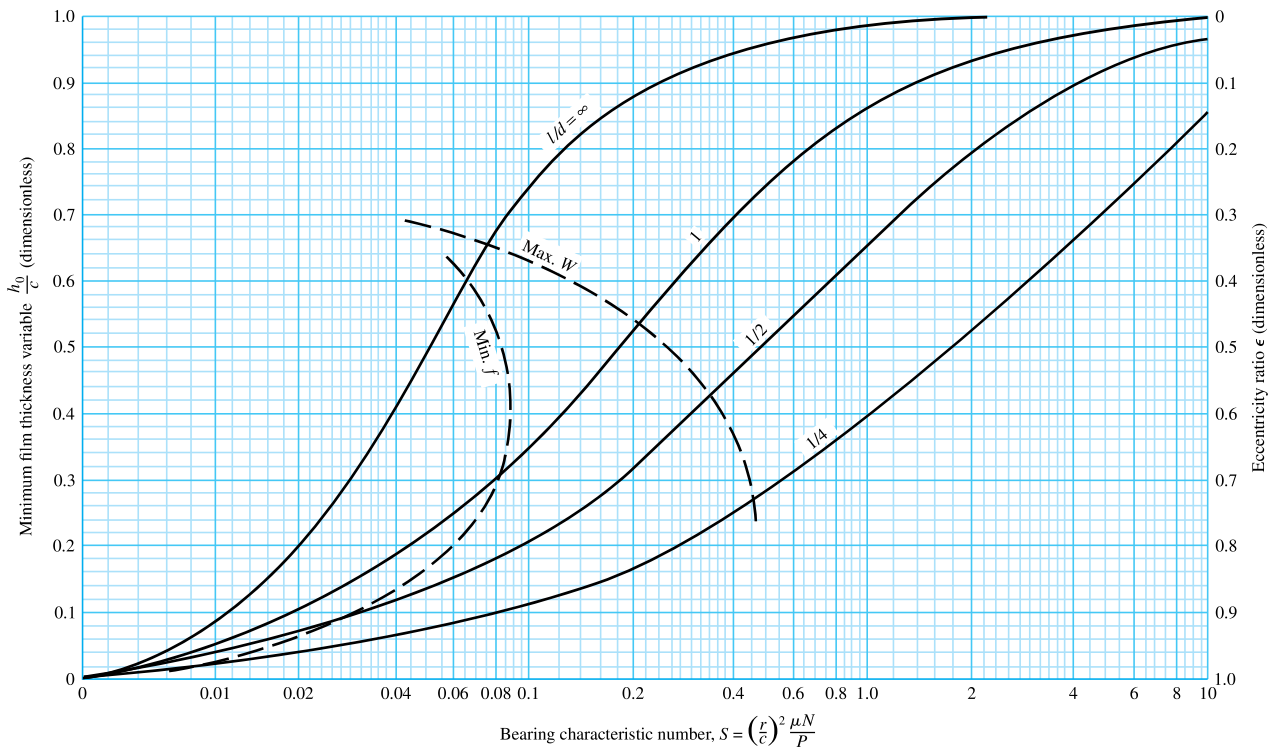
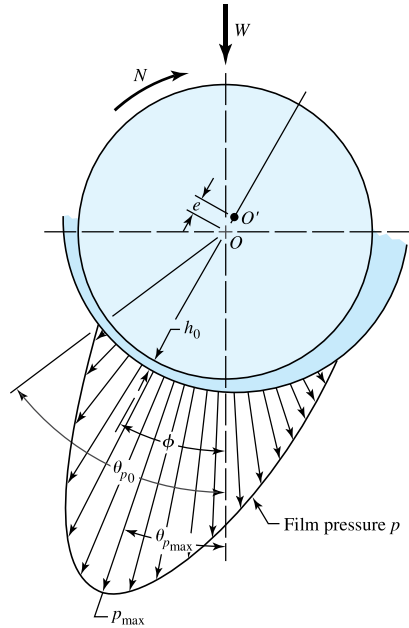
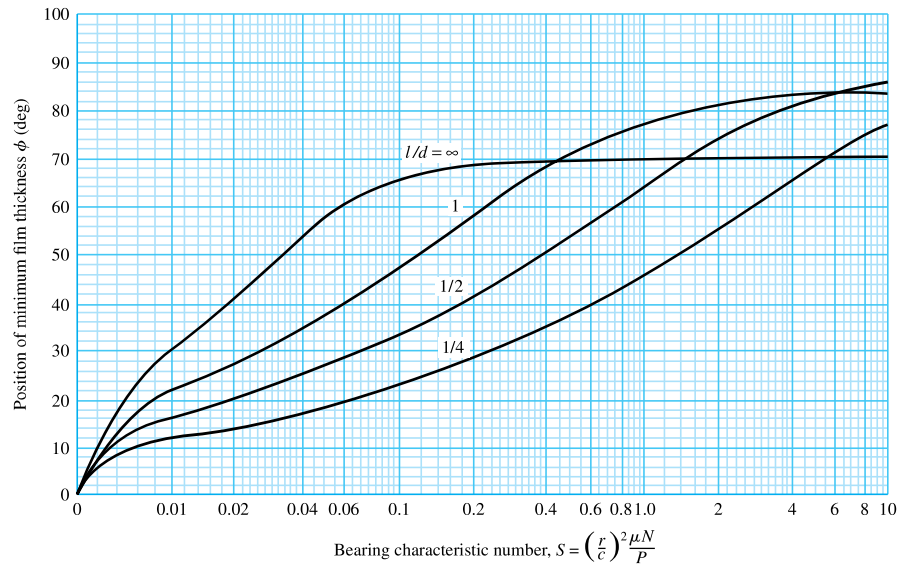
**Figure 12-16**

Chart for minimum film thickness variable and eccentricity ratio. The left boundary of the zone defines the optimal h_0 for minimum friction; the right boundary is optimum h_0 for load. (Raimondi and Boyd.)

Figure 12-17

Chart for determining the position of the minimum film thickness h_0 . (Raimondi and Boyd.)



The remaining charts from Raimondi and Boyd relate several variables to the Sommerfeld number. These variables are

- Minimum film thickness (Figs. 12-16 and 12-17)
- Coefficient of friction (Fig. 12-18)
- Lubricant flow (Figs. 12-19 and 12-20)
- Film pressure (Figs. 12-21 and 12-22)

Figure 12-15 shows the notation used for the variables. We will describe the use of these curves in a series of four examples using the same set of given parameters.

Minimum Film Thickness

In Fig. 12-16, the *minimum film thickness* variable h_0/c and *eccentricity ratio* $\epsilon = e/c$ are plotted against the Sommerfeld number S with contours for various values of l/d . The corresponding angular position of the minimum film thickness is found in Fig. 12-17.

EXAMPLE 12-1

Determine h_0 and e using the following given parameters: $\mu = 4 \mu\text{reyn}$, $N = 30 \text{ rev/s}$, $W = 500 \text{ lbf}$ (bearing load), $r = 0.75 \text{ in}$, $c = 0.0015 \text{ in}$, and $l = 1.5 \text{ in}$.

Solution

The nominal bearing pressure (in projected area of the journal) is

$$P = \frac{W}{2rl} = \frac{500}{2(0.75)1.5} = 222 \text{ psi}$$

The Sommerfeld number is, from Eq. (12-7), where $N = N_j = 30 \text{ rev/s}$,

$$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu N}{P}\right) = \left(\frac{0.75}{0.0015}\right)^2 \left[\frac{4(10^{-6})30}{222}\right] = 0.135$$

Also, $l/d = 1.50/[2(0.75)] = 1$. Entering Fig. 12-16 with $S = 0.135$ and $l/d = 1$ gives $h_0/c = 0.42$ and $\epsilon = 0.58$. The quantity h_0/c is called the *minimum film thickness*

variable. Since $c = 0.0015$ in, the minimum film thickness h_0 is

$$h_0 = 0.42(0.0015) = 0.00063 \text{ in}$$

We can find the angular location ϕ of the minimum film thickness from the chart of Fig. 12-17. Entering with $S = 0.135$ and $l/d = 1$ gives $\phi = 53^\circ$.

The eccentricity ratio is $\epsilon = e/c = 0.58$. This means the eccentricity e is

$$e = 0.58(0.0015) = 0.00087 \text{ in}$$

Note that if the journal is centered in the bushing, $e = 0$ and $h_0 = c$, corresponding to a very light (zero) load. Since $e = 0$, $\epsilon = 0$. As the load is increased the journal displaces downward; the limiting position is reached when $h_0 = 0$ and $e = c$, that is, when the journal touches the bushing. For this condition the eccentricity ratio is unity. Since $h_0 = c - e$, dividing both sides by c , we have

$$\frac{h_0}{c} = 1 - \epsilon$$

Design optima are sometimes *maximum load*, which is a load-carrying characteristic of the bearing, and sometimes *minimum parasitic power loss* or *minimum coefficient of friction*. Dashed lines appear on Fig. 12-16 for maximum load and minimum coefficient of friction, so you can easily favor one of maximum load or minimum coefficient of friction, but not both. The zone between the two dashed-line contours might be considered a desirable location for a design point.

Coefficient of Friction

The friction chart, Fig. 12-18, has the *friction variable* $(r/c)f$ plotted against Sommerfeld number S with contours for various values of the l/d ratio.

EXAMPLE 12-2

Using the parameters given in Ex. 12-1, determine the coefficient of friction, the torque to overcome friction, and the power loss to friction.

Solution

We enter Fig. 12-18 with $S = 0.135$ and $l/d = 1$ and find $(r/c)f = 3.50$. The coefficient of friction f is

$$f = 3.50 c/r = 3.50(0.0015/0.75) = 0.0070$$

The friction torque on the journal is

$$T = fWr = 0.007(500)0.75 = 2.62 \text{ lbf} \cdot \text{in}$$

The power loss in horsepower is

$$(hp)_{\text{loss}} = \frac{TN}{1050} = \frac{2.62(30)}{1050} = 0.075 \text{ hp}$$

or, expressed in Btu/s,

$$H = \frac{2\pi TN}{778(12)} = \frac{2\pi(2.62)(30)}{778(12)} = 0.0529 \text{ Btu/s}$$

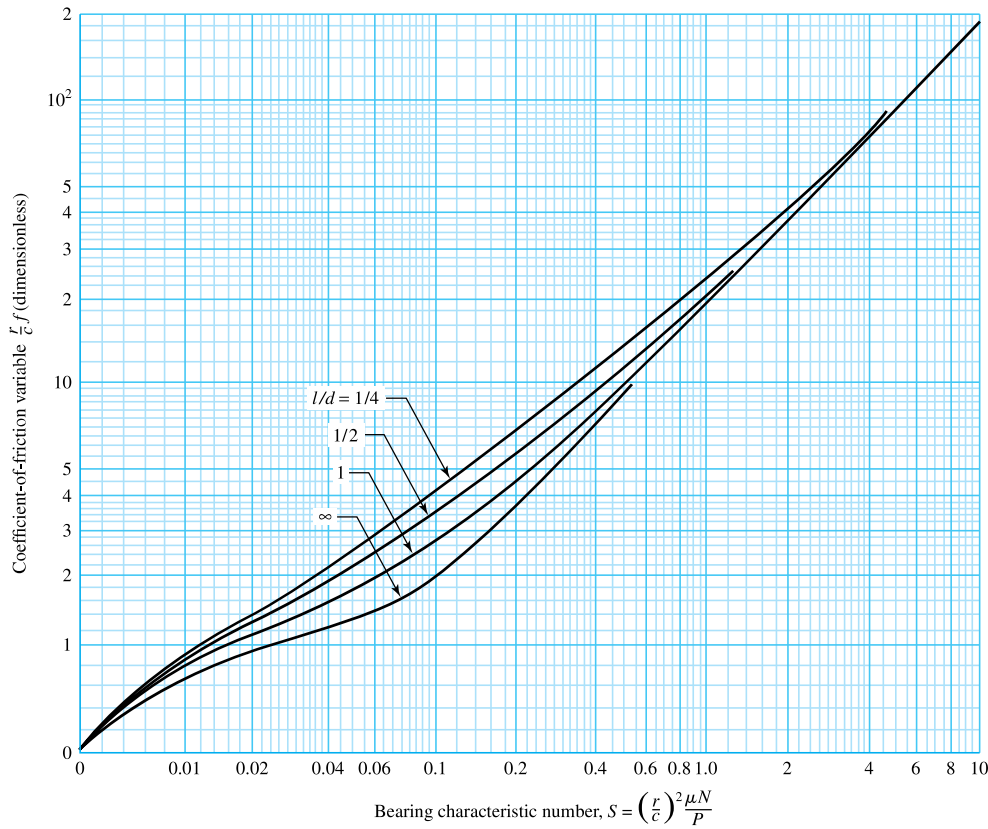


Figure 12-18

Chart for coefficient-of-friction variable; note that Petroff's equation is the asymptote. (Raimondi and Boyd.)

Lubricant Flow

Figures 12-19 and 12-20 are used to determine the lubricant flow and side flow.

EXAMPLE 12-3

Continuing with the parameters of Ex. 12-1, determine the total volumetric flow rate Q and the side flow rate Q_s .

Solution

To estimate the lubricant flow, enter Fig. 12-19 with $S = 0.135$ and $l/d = 1$ to obtain $Q/(rcNl) = 4.28$. The total volumetric flow rate is

$$Q = 4.58rcNl = 4.28(0.75)0.0015(30)1.5 = 0.217 \text{ in}^3/\text{s}$$

From Fig. 12-20 we find the flow ratio $Q_s/Q = 0.655$ and Q_s is

$$Q_s = 0.655Q = 0.655(0.217) = 0.142 \text{ in}^3/\text{s}$$

Figure 12-19

Chart for flow variable.
Note: Not for pressure-fed bearings. (Raimondi and Boyd.)

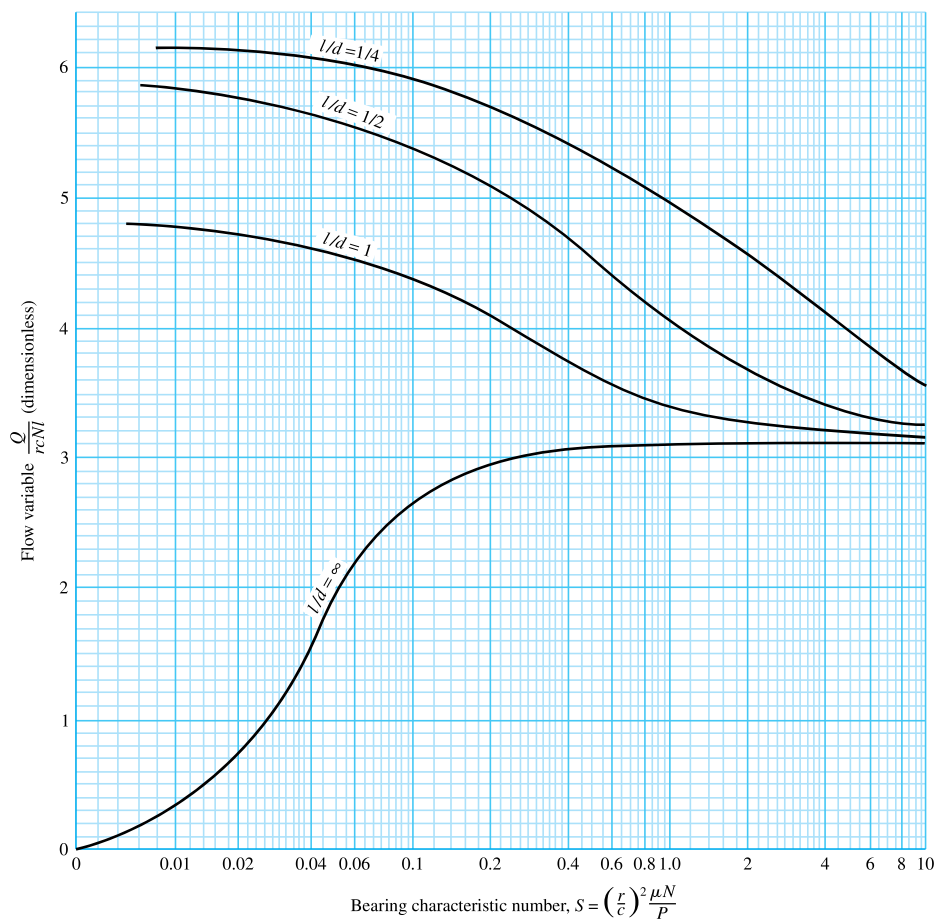
**Figure 12-20**

Chart for determining the ratio of side flow to total flow.
 (Raimondi and Boyd.)

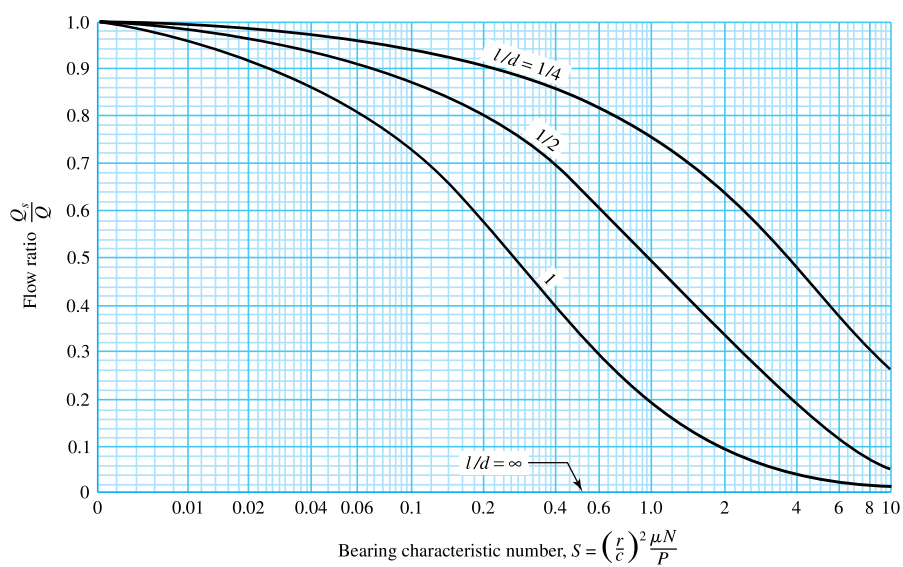
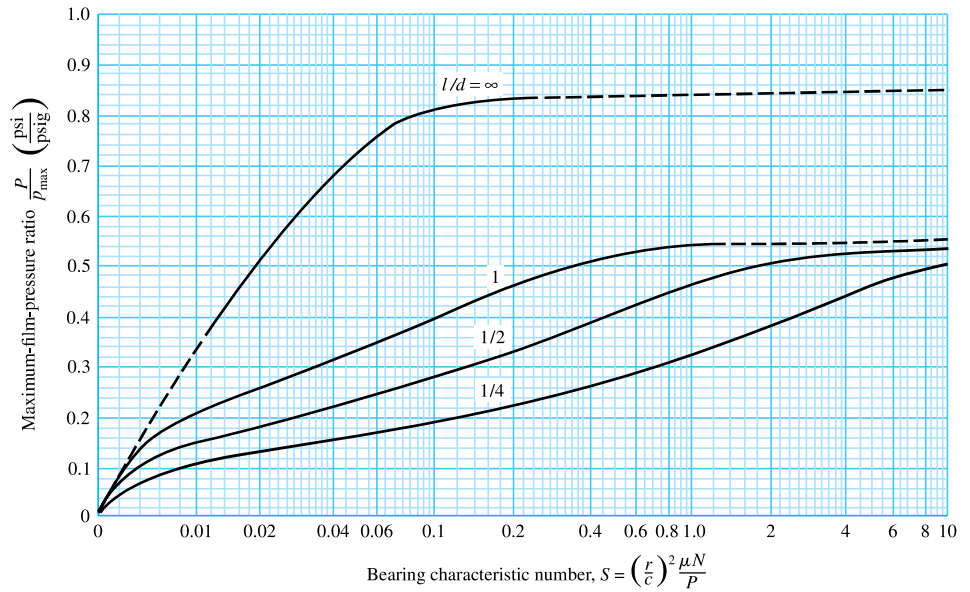


Figure 12-21

Chart for determining the maximum film pressure.

Note: Not for pressure-fed bearings. (Raimondi and Boyd.)



The side leakage Q_s is from the lower part of the bearing, where the internal pressure is above atmospheric pressure. The leakage forms a fillet at the journal-bushing external junction, and it is carried by journal motion to the top of the bushing, where the internal pressure is below atmospheric pressure and the gap is much larger, to be “sucked in” and returned to the lubricant sump. That portion of side leakage that leaks away from the bearing has to be made up by adding oil to the bearing sump periodically by maintenance personnel.

Film Pressure

The maximum pressure developed in the film can be estimated by finding the pressure ratio P/p_{\max} from the chart in Fig. 12-21. The locations where the terminating and maximum pressures occur, as defined in Fig 12-15, are determined from Fig. 12-22.

EXAMPLE 12-4

Using the parameters given in Ex. 12-1, determine the maximum film pressure and the locations of the maximum and terminating pressures.

Solution

Entering Fig. 12-21 with $S = 0.135$ and $l/d = 1$, we find $P/p_{\max} = 0.42$. The maximum pressure p_{\max} is therefore

$$p_{\max} = \frac{P}{0.42} = \frac{222}{0.42} = 529 \text{ psi}$$

With $S = 0.135$ and $l/d = 1$, from Fig. 12-22, $\theta_{p_{\max}} = 18.5^\circ$ and the terminating position θ_{p_0} is 75° .

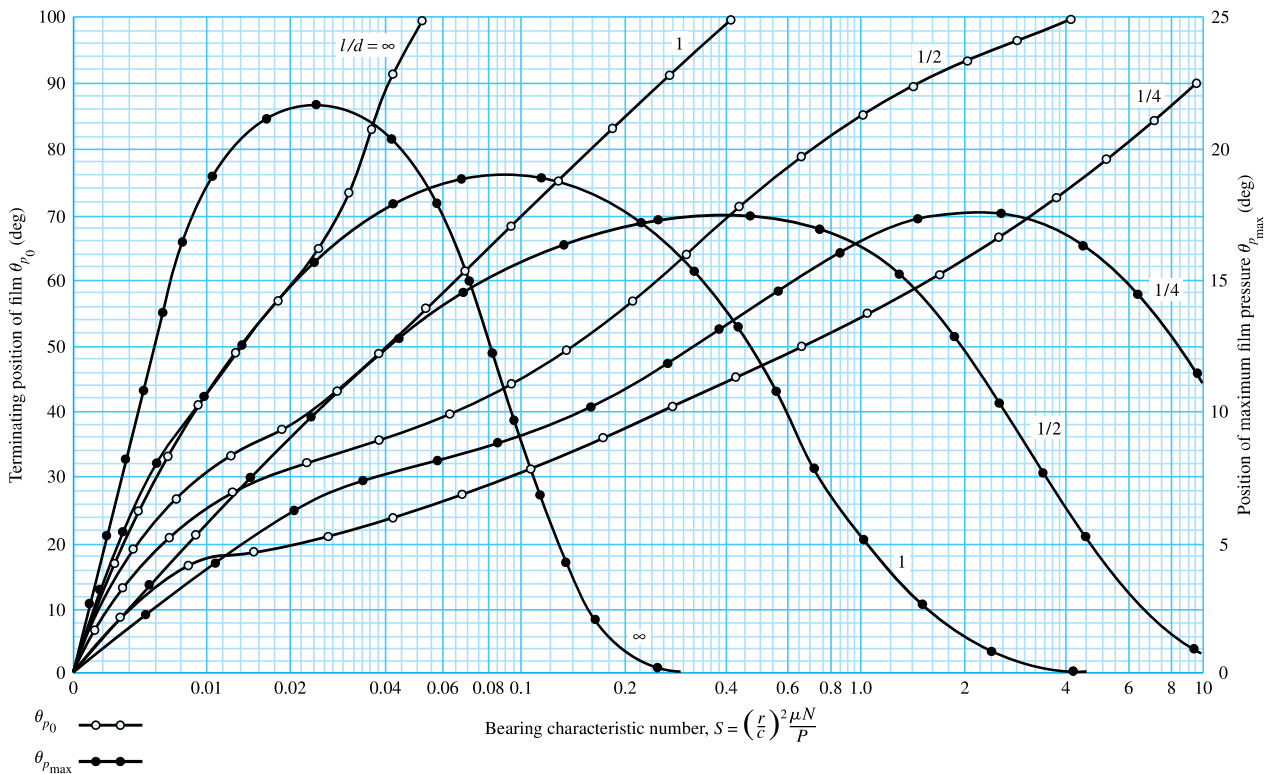


Figure 12-22

Chart for finding the terminating position of the lubricant film and the position of maximum film pressure. (Raimondi and Boyd.)

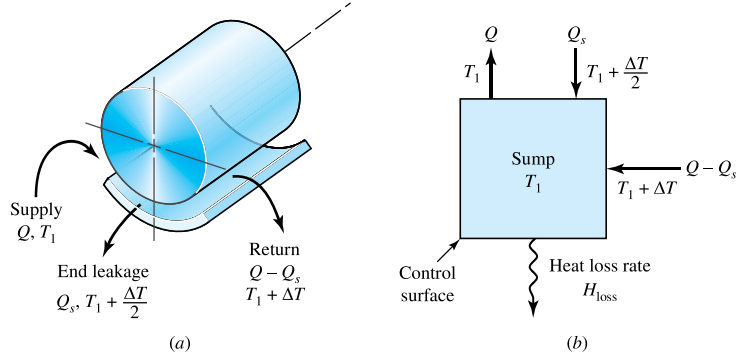
Examples 12–1 to 12–4 demonstrate how the Raimondi and Boyd charts are used. It should be clear that we do not have journal-bearing parametric relations as equations, but in the form of charts. Moreover, the examples were simple because the steady-state equivalent viscosity was given. We will now show how the average film temperature (and the corresponding viscosity) is found from energy considerations.

Lubricant Temperature Rise

The temperature of the lubricant rises until the rate at which work is done by the journal on the film through fluid shear is the same as the rate at which heat is transferred to the greater surroundings. The specific arrangement of the bearing plumbing affects the quantitative relationships. See Fig. 12–23. A lubricant sump (internal or external to the bearing housing) supplies lubricant at sump temperature T_s to the bearing annulus at temperature $T_s = T_1$. The lubricant passes once around the bushing and is delivered at a higher lubricant temperature $T_1 + \Delta T$ to the sump. Some of the lubricant leaks out of the bearing at a mixing-cup temperature of $T_1 + \Delta T/2$ and is returned to the sump. The sump may be a keyway-like groove in the bearing cap or a larger chamber up to half the bearing circumference. It can occupy “all” of the bearing cap of a split bearing. In such a bearing the side leakage occurs from the lower portion and is sucked back in, into the ruptured film arc. The sump could be well removed from the journal-bushing interface.

Figure 12-23

Schematic of a journal bearing with an external sump with cooling; lubricant makes one pass before returning to the sump.



Let

Q = volumetric oil-flow rate into the bearing, in^3/s

Q_s = volumetric side-flow leakage rate out of the bearing and to the sump, in^3/s

$Q - Q_s$ = volumetric oil-flow discharge from annulus to sump, in^3/s

T_1 = oil inlet temperature (equal to sump temperature T_s), $^{\circ}\text{F}$

ΔT = temperature rise in oil between inlet and outlet, $^{\circ}\text{F}$

ρ = lubricant density, lbm/in^3

C_p = specific heat capacity of lubricant, $\text{Btu}/(\text{lbm} \cdot ^{\circ}\text{F})$

J = Joulean heat equivalent, $\text{in} \cdot \text{lbf}/\text{Btu}$

H = heat rate, Btu/s

Using the sump as a control region, we can write an enthalpy balance. Using T_1 as the datum temperature gives

$$H_{\text{loss}} = \rho C_p Q_s \Delta T / 2 + \rho C_p (Q - Q_s) \Delta T = \rho C_p Q \Delta T \left(1 - 0.5 \frac{Q_s}{Q} \right) \quad (a)$$

The thermal energy loss at steady state H_{loss} is equal to the rate the journal does work on the film is $H_{\text{loss}} = \dot{W} = 2\pi TN/J$. The torque $T = fWr$, the load in terms of pressure is $W = 2Pr$, and multiplying numerator and denominator by the clearance c gives

$$H_{\text{loss}} = \frac{4\pi PrlNc}{J} \frac{rf}{c} \quad (b)$$

Equating Eqs. (a) and (b) and rearranging results in

$$\frac{J\rho C_p \Delta T}{4\pi P} = \frac{rf/c}{(1 - 0.5Q_s/Q)[Q/(rcNl)]} \quad (c)$$

For common petroleum lubricants $\rho = 0.0311 \text{ lbm}/\text{in}^3$, $C_p = 0.42 \text{ Btu}/(\text{lbm} \cdot ^{\circ}\text{F})$, and $J = 778(12) = 9336 \text{ in} \cdot \text{lbf}/\text{Btu}$; therefore the left term of Eq. (c) is

$$\frac{J\rho C_p \Delta T}{4\pi P} = \frac{9336(0.0311)(0.42)\Delta T_F}{4\pi P_{\text{psi}}} = 9.70 \frac{\Delta T_F}{P_{\text{psi}}}$$

thus

$$\frac{9.70\Delta T_F}{P_{\text{psi}}} = \frac{rf/c}{(1 - 0.5Q_s/Q)[Q/(rcNl)]} \quad (12-15)$$

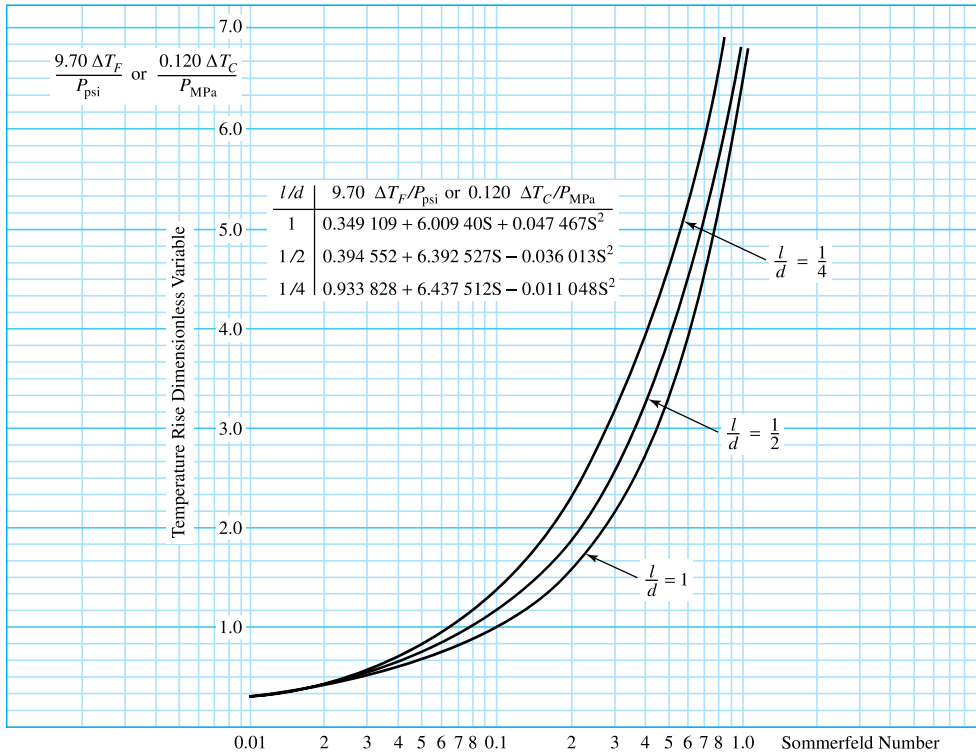


Figure 12-24

Figures 12-18, 12-19, and 12-20 combined to reduce iterative table look-up. (Source: Chart based on work of Raimondi and Boyd boundary condition (2), i.e., no negative lubricant pressure developed. Chart is for full journal bearing using single lubricant pass, side flow emerges with temperature rise $\Delta T/2$, thru flow emerges with temperature rise ΔT , and entire flow is supplied at datum sump temperature.)

where ΔT_F is the temperature rise in $^{\circ}\text{F}$ and P_{psi} is the bearing pressure in psi. The right side of Eq. (12-15) can be evaluated from Figs. 12-18, 12-19, and 12-20 for various Sommerfeld numbers and l/d ratios to give Fig. 12-24. It is easy to show that the left side of Eq. (12-15) can be expressed as $0.120\Delta T_C/P_{\text{MPa}}$ where ΔT_C is expressed in $^{\circ}\text{C}$ and the pressure P_{MPa} is expressed in MPa. The ordinate in Fig. 12-24 is either $9.70\Delta T_F/P_{\text{psi}}$ or $0.120\Delta T_C/P_{\text{MPa}}$, which is not surprising since both are dimensionless in proper units and *identical in magnitude*. Since solutions to bearing problems involve iteration and reading many graphs can introduce errors, Fig. 12-24 reduces three graphs to one, a step in the proper direction.

Interpolation

For l/d ratios other than the ones given in the charts, Raimondi and Boyd have provided the following interpolation equation

$$y = \frac{1}{(l/d)^3} \left[-\frac{1}{8} \left(1 - \frac{l}{d}\right) \left(1 - 2\frac{l}{d}\right) \left(1 - 4\frac{l}{d}\right) y_{\infty} + \frac{1}{3} \left(1 - 2\frac{l}{d}\right) \left(1 - 4\frac{l}{d}\right) y_1 \right. \\ \left. - \frac{1}{4} \left(1 - \frac{l}{d}\right) \left(1 - 4\frac{l}{d}\right) y_{1/2} + \frac{1}{24} \left(1 - \frac{l}{d}\right) \left(1 - 2\frac{l}{d}\right) y_{1/4} \right] \quad (12-16)$$

where y is the desired variable within the interval $\infty > l/d > \frac{1}{4}$ and $y_\infty, y_1, y_{1/2}, y_{1/4}$ are the variables corresponding to l/d ratios of $\infty, 1, \frac{1}{2},$ and $\frac{1}{4}$, respectively.

12-9 Steady-State Conditions in Self-Contained Bearings

The case in which the lubricant carries away all of the enthalpy increase from the journal-bushing pair has already been discussed. Bearings in which the warm lubricant stays within the bearing housing will now be addressed. These bearings are called *self-contained* bearings because the lubricant sump is within the bearing housing and the lubricant is cooled within the housing. These bearings are described as *pillow-block* or *pedestal* bearings. They find use on fans, blowers, pumps, and motors, for example. Integral to design considerations for these bearings is dissipating heat from the bearing housing to the surroundings at the same rate that enthalpy is being generated within the fluid film.

In a self-contained bearing the sump can be positioned as a keywaylike cavity in the bushing, the ends of the cavity not penetrating the end planes of the bushing. Film oil exits the annulus at about one-half of the relative peripheral speeds of the journal and bushing and slowly tumbles the sump lubricant, mixing with the sump contents. Since the film in the top “half” of the cap has cavitated, it contributes essentially nothing to the support of the load, but it does contribute friction. Bearing caps are in use in which the “keyway” sump is expanded peripherally to encompass the top half of the bearing. This reduces friction for the same load, but the included angle β of the bearing has been reduced to 180° . Charts for this case were included in the Raimondi and Boyd paper.

The heat given up by the bearing housing may be estimated from the equation

$$H_{\text{loss}} = \bar{h}_{\text{CR}} A (T_b - T_\infty) \quad (12-17)$$

where H_{loss} = heat dissipated, Btu/h

\bar{h}_{CR} = combined overall coefficient of radiation and convection heat transfer, Btu/(h · ft² · °F)

A = surface area of bearing housing, ft²

T_b = surface temperature of the housing, °F

T_∞ = ambient temperature, °F

The overall coefficient \bar{h}_{CR} depends on the material, surface coating, geometry, even the roughness, the temperature difference between the housing and surrounding objects, and air velocity. After Karelitz,¹⁰ and others, in ordinary industrial environments, the overall coefficient \bar{h}_{CR} can be treated as a constant. Some representative values are

$$\bar{h}_{\text{CR}} = \begin{cases} 2 \text{ Btu}/(\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}) & \text{for still air} \\ 2.7 \text{ Btu}/(\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}) & \text{for shaft-stirred air} \\ 5.9 \text{ Btu}/(\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}) & \text{for air moving at 500 ft/min} \end{cases} \quad (12-18)$$

An expression similar to Eq. (12-17) can be written for the temperature difference $T_f - T_b$ between the lubricant film and the housing surface. This is possible because the bushing and housing are metal and very nearly isothermal. If one defines \bar{T}_f as the *average* film temperature (halfway between the lubricant inlet temperature T_s and

¹⁰G. B. Karelitz, “Heat Dissipation in Self-Contained Bearings,” *Trans. ASME*, Vol. 64, 1942, p. 463;

D. C. Lemmon and E. R. Booser, “Bearing Oil-Ring Performance,” *Trans. ASME, J. Bas. Engin.*, Vol. 88, 1960, p. 327.

Table 12-2

Lubrication System	Conditions	Range of α
Oil ring	Moving air	1–2
	Still air	$\frac{1}{2}$ –1
Oil bath	Moving air	$\frac{1}{2}$ –1
	Still air	$\frac{1}{5}$ – $\frac{2}{5}$

the outlet temperature $T_s + \Delta T$), then the following proportionality has been observed between $\bar{T}_f - T_b$ and the difference between the housing surface temperature and the ambient temperature, $T_b - T_\infty$:

$$\bar{T}_f - T_b = \alpha(T_b - T_\infty) \quad (a)$$

where \bar{T}_f is the average film temperature and α is a constant depending on the lubrication scheme and the bearing housing geometry. Equation (a) may be used to estimate the bearing housing temperature. Table 12-2 provides some guidance concerning suitable values of α . The work of Karelitz allows the broadening of the application of the charts of Raimondi and Boyd, to be applied to a variety of bearings beyond the natural circulation pillow-block bearing.

Solving Eq. (a) for T_b and substituting into Eq. (12-17) gives the bearing heat loss rate to the surroundings as

$$H_{\text{loss}} = \frac{\dot{h}_{\text{CR}} A}{1 + \alpha} (\bar{T}_f - T_\infty) \quad (12-19a)$$

and rewriting Eq. (a) gives

$$T_b = \frac{\bar{T}_f + \alpha T_\infty}{1 + \alpha} \quad (12-19b)$$

In beginning a steady-state analysis the average film temperature is unknown, hence the viscosity of the lubricant in a self-contained bearing is unknown. Finding the equilibrium temperatures is an iterative process wherein a trial average film temperature (and the corresponding viscosity) is used to compare the heat generation rate and the heat loss rate. An adjustment is made to bring these two heat rates into agreement. This can be done on paper with a tabular array to help adjust \bar{T}_f to achieve equality between heat generation and loss rates. A root-finding algorithm can be used. Even a simple one can be programmed for a digital computer.

Because of the shearing action there is a uniformly distributed energy release in the lubricant that heats the lubricant as it works its way around the bearing. The temperature is uniform in the radial direction but increases from the sump temperature T_s by an amount ΔT during the lubricant pass. The exiting lubricant mixes with the sump contents, being cooled to sump temperature. The lubricant in the sump is cooled because the bushing and housing metal are at a nearly uniform lower temperature because of heat losses by convection and radiation to the surroundings at ambient temperature T_∞ . In the usual configurations of such bearings, the bushing and housing metal temperature is approximately midway between the average film temperature $\bar{T}_f = T_s + \Delta T/2$ and the ambient temperature T_∞ . The heat generation rate H_{gen} , at steady state, is equal to the work rate from the frictional torque T . Expressing this in Btu/h requires the conversion constants 2545 Btu/(hp · h) and 1050 (lbf · in)(rev/s)/hp results

in $H_{\text{gen}} = 2545 \text{ TN}/1050$. Then from Eq. (b), Sec. 12-3, the torque is $T = 4\pi^2 r^3 l \mu / c$, resulting in

$$H_{\text{gen}} = \frac{2545}{1050} \frac{4\pi^2 r^3 l \mu N}{c} = \frac{95.69 \mu N^2 l r^3}{c} \quad (b)$$

Equating this to Eq. (12-19a) and solving for \bar{T}_f gives

$$\bar{T}_f = T_{\infty} + 95.69(1 + \alpha) \frac{\mu N^2 l r^3}{\hbar_{\text{CR}} A c} \quad (12-20)$$

EXAMPLE 12-5

Consider a pillow-block bearing with a keyway sump, whose journal rotates at 900 rev/min in shaft-stirred air at 70°F with $\alpha = 1$. The lateral area of the bearing is 40 in². The lubricant is SAE grade 20 oil. The gravity radial load is 100 lbf and the l/d ratio is unity. The bearing has a journal diameter of 2.000 + 0.000/−0.002 in, a bushing bore of 2.002 + 0.004/−0.000 in. For a minimum clearance assembly estimate the steady-state temperatures as well as the minimum film thickness and coefficient of friction.

Solution

The minimum radial clearance, c_{min} , is

$$c_{\text{min}} = \frac{2.002 - 2.000}{2} = 0.001 \text{ in}$$

$$P = \frac{W}{ld} = \frac{100}{(2)2} = 25 \text{ psi}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{1}{0.001}\right)^2 \frac{\mu'(15)}{10^6(25)} = 0.6 \mu'$$

where μ' is viscosity in μreyn . The friction horsepower loss, $(\text{hp})_f$, is found as follows:

$$(\text{hp})_f = \frac{f W r N}{1050} = \frac{W N c}{1050} \frac{f r}{c} = \frac{100(900/60)0.001}{1050} \frac{f r}{c} = 0.001429 \frac{f r}{c} \text{ hp}$$

The heat generation rate H_{gen} , in Btu/h, is

$$H_{\text{gen}} = 2545(\text{hp})_f = 2545(0.001429) f r / c = 3.637 f r / c \text{ Btu/h}$$

From Eq. (12-19a) with $\hbar_{\text{CR}} = 2.7 \text{ Btu}/(\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F})$, the rate of heat loss to the environment H_{loss} is

$$H_{\text{loss}} = \frac{\hbar_{\text{CR}} A}{\alpha + 1} (\bar{T}_f - 70) = \frac{2.7(40/144)}{(1 + 1)} (\bar{T}_f - 70) = 0.375(\bar{T}_f - 70) \text{ Btu/h}$$

Build a table as follows for trial values of \bar{T}_f of 190 and 195°F:

Trial	\bar{T}_f	μ'	S	$f r / c$	H_{gen}	H_{loss}
190		1.15	0.69	13.6	49.5	45.0
195		1.03	0.62	12.2	44.4	46.9

The temperature at which $H_{\text{gen}} = H_{\text{loss}} = 46.3 \text{ Btu/h}$ is 193.4°F . Rounding \bar{T}_f to 193°F we find $\mu' = 1.08 \text{ } \mu\text{reyn}$ and $S = 0.6(1.08) = 0.65$. From Fig. 12-24, $9.70\Delta T_F/P = 4.25^\circ\text{F/psi}$ and thus

$$\Delta T_F = 4.25P/9.70 = 4.25(25)/9.70 = 11.0^\circ\text{F}$$

$$T_1 = T_s = \bar{T}_f - \Delta T/2 = 193 - 11/2 = 187.5^\circ\text{F}$$

$$T_{\text{max}} = T_1 + \Delta T_F = 187.5 + 11 = 198.5^\circ\text{F}$$

From Eq. (12-19b)

$$T_b = \frac{T_f + \alpha T_\infty}{1 + \alpha} = \frac{193 + (1)70}{1 + 1} = 131.5^\circ\text{F}$$

with $S = 0.65$, the minimum film thickness from Fig. 12-16 is

$$h_0 = \frac{h_0}{c} c = 0.79(0.001) = 0.00079 \text{ in}$$

The coefficient of friction from Fig. 12-18 is

$$f = \frac{fr}{c} \frac{c}{r} = 12.8 \frac{0.001}{1} = 0.0128$$

The parasitic friction torque T is

$$T = fWr = 0.0128(100)(1) = 1.28 \text{ lbf} \cdot \text{in}$$

12-10 Clearance

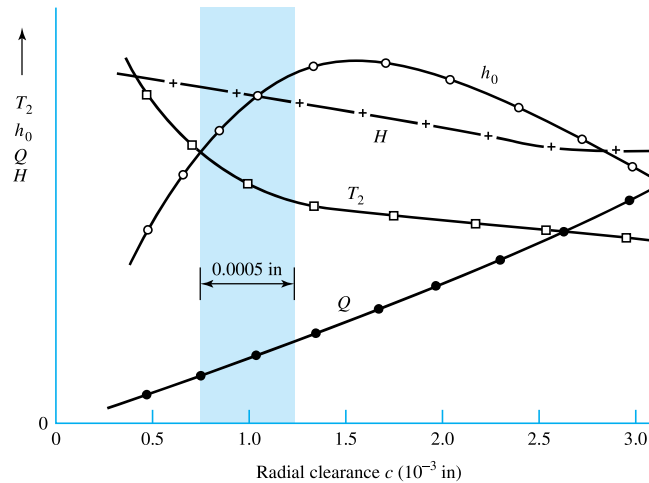
In designing a journal bearing for thick-film lubrication, the engineer must select the grade of oil to be used, together with suitable values for P , N , r , c , and l . A poor selection of these or inadequate control of them during manufacture or in use may result in a film that is too thin, so that the oil flow is insufficient, causing the bearing to overheat and, eventually, fail. Furthermore, the radial clearance c is difficult to hold accurate during manufacture, and it may increase because of wear. What is the effect of an entire range of radial clearances, expected in manufacture, and what will happen to the bearing performance if c increases because of wear? Most of these questions can be answered and the design optimized by plotting curves of the performance as functions of the quantities over which the designer has control.

Figure 12-25 shows the results obtained when the performance of a particular bearing is calculated for a whole range of radial clearances and is plotted with clearance as the independent variable. The bearing used for this graph is the one of Examples 12-1 to 12-4 with SAE 20 oil at an inlet temperature of 100°F . The graph shows that if the clearance is too tight, the temperature will be too high and the minimum film thickness too low. High temperatures may cause the bearing to fail by fatigue. If the oil film is too thin, dirt particles may be unable to pass without scoring or may embed themselves in the bearing. In either event, there will be excessive wear and friction, resulting in high temperatures and possible seizing.

To investigate the problem in more detail, Table 12-3 was prepared using the two types of preferred running fits that seem to be most useful for journal-bearing design

Figure 12-25

A plot of some performance characteristics of the bearing of Exs. 12-1 to 12-4 for radial clearances of 0.0005 to 0.003 in. The bearing outlet temperature is designated T_2 . New bearings should be designed for the shaded zone, because wear will move the operating point to the right.

**Table 12-3**

Maximum, Minimum, and Average Clearances for 1.5-in-Diameter Journal Bearings Based on Type of Fit

Type of Fit	Symbol	Clearance c , in		
		Maximum	Average	Minimum
Close-running	H8/f7	0.001 75	0.001 125	0.000 5
Free-running	H9/d9	0.003 95	0.002 75	0.001 55

Table 12-4

Performance of 1.5-in-Diameter Journal Bearing with Various Clearances. (SAE 20 Lubricant, $T_1 = 100^\circ\text{F}$, $N = 30$ r/s, $W = 500$ lbf, $L = 1.5$ in)

c , in	T_2 , $^\circ\text{F}$	h_0 , in	f	Q , in^3/s	H , Btu/s
0.000 5	226	0.000 38	0.011 3	0.061	0.086
0.001 125	142	0.000 65	0.009 0	0.153	0.068
0.001 55	133	0.000 77	0.008 7	0.218	0.066
0.001 75	128	0.000 76	0.008 4	0.252	0.064
0.002 75	118	0.000 73	0.007 9	0.419	0.060
0.003 95	113	0.000 69	0.007 7	0.617	0.059

(see Table 7-9), p. 389. The results shown in Table 12-3 were obtained by using Eqs. (7-36) and (7-37) of Sec. 7-8. Notice that there is a slight overlap, but the range of clearances for the free-running fit is about twice that of the close-running fit.

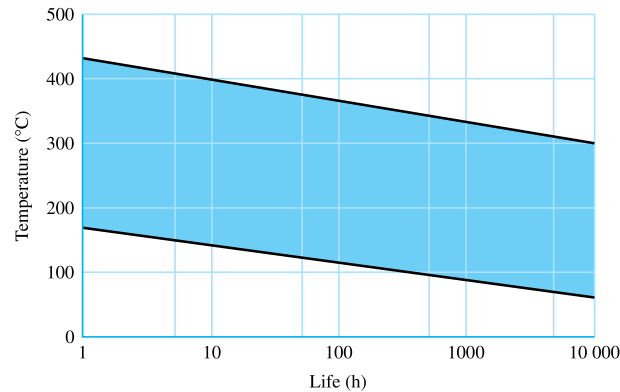
The six clearances of Table 12-3 were used in a computer program to obtain the numerical results shown in Table 12-4. These conform to the results of Fig. 12-25, too. Both the table and the figure show that a tight clearance results in a high temperature. Figure 12-26 can be used to estimate an upper temperature limit when the characteristics of the application are known.

It would seem that a large clearance will permit the dirt particles to pass through and also will permit a large flow of oil, as indicated in Table 12-4. This lowers the temperature and increases the life of the bearing. However, if the clearance becomes

Figure 12-26

Temperature limits for mineral oils. The lower limit is for oils containing antioxidants and applies when oxygen supply is unlimited. The upper limit applies when insignificant oxygen is present. The life in the shaded zone depends on the amount of oxygen and catalysts present.

(Source: M. J. Neale (ed.), *Tribology Handbook, Section B1*, Newnes-Butterworth, London, 1975.)



too large, the bearing becomes noisy and the minimum film thickness begins to decrease again.

In between these two limitations there exists a rather large range of clearances that will result in satisfactory bearing performance.

When both the production tolerance and the future wear on the bearing are considered, it is seen, from Fig. 12-25, that the best compromise is a clearance range slightly to the left of the top of the minimum-film-thickness curve. In this way, future wear will move the operating point to the right and increase the film thickness and decrease the operating temperature.

12-11 Pressure-Fed Bearings

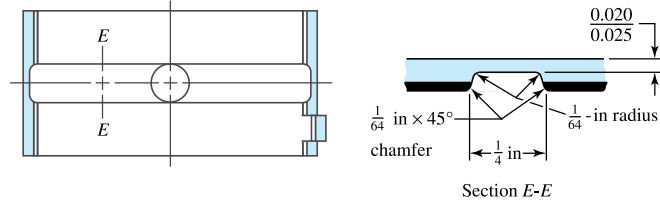
The load-carrying capacity of self-contained natural-circulating journal bearings is quite restricted. The factor limiting better performance is the heat-dissipation capability of the bearing. A first thought of a way to increase heat dissipation is to cool the sump with an external fluid such as water. The high-temperature problem is in the film where the heat is generated but cooling is not possible in the film until later. This does not protect against exceeding the maximum allowable temperature of the lubricant. A second alternative is to reduce the *temperature rise* in the film by dramatically increasing the rate of lubricant flow. The lubricant itself is reducing the temperature rise. A water-cooled sump may still be in the picture. To increase lubricant flow, an external pump must be used with lubricant supplied at pressures of tens of pounds per square inch gage. Because the lubricant is supplied to the bearing under pressure, such bearings are called *pressure-fed bearings*.

To force a greater flow through the bearing and thus obtain an increased cooling effect, a common practice is to use a circumferential groove at the center of the bearing, with an oil-supply hole located opposite the load-bearing zone. Such a bearing is shown in Fig. 12-27. The effect of the groove is to create two half-bearings, each having a smaller l/d ratio than the original. The groove divides the pressure-distribution curve into two lobes and reduces the minimum film thickness, but it has wide acceptance among lubrication engineers because such bearings carry more load without overheating.

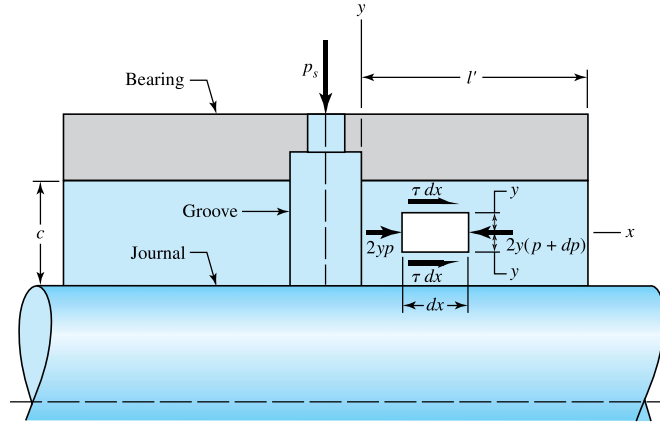
To set up a method of solution for oil flow, we shall assume a groove ample enough that the pressure drop in the groove itself is small. Initially we will neglect eccentricity and then apply a correction factor for this condition. The oil flow, then, is the amount that flows out of the two halves of the bearing in the direction of the concentric shaft. If we neglect the rotation of the shaft, the flow of the lubricant is

Figure 12-27

Centrally located full annular groove. (Courtesy of the Cleveland Graphite Bronze Company, Division of Clevite Corporation.)

**Figure 12-28**

Flow of lubricant from a pressure-fed bearing having a central annular groove.



caused by the supply pressure p_s , shown in Fig. 12-28. Laminar flow is assumed, with the pressure varying linearly from $p = p_s$ at $x = 0$, to $p = 0$ at $x = l'$. Consider the static equilibrium of an element of thickness dx , height $2y$, and unit depth. Note particularly that the origin of the reference system has been chosen at the midpoint of the clearance space and symmetry about the x axis is implied with the shear stresses τ being equal on the top and bottom surfaces. The equilibrium equation in the x direction is

$$-2y(p + dp) + 2yp + 2\tau dx = 0 \quad (a)$$

Expanding and canceling terms, we find that

$$\tau = y \frac{dp}{dx} \quad (b)$$

Newton's equation for viscous flow [Eq. (12-1)] is

$$\tau = \mu \frac{du}{dy} \quad (c)$$

Now eliminating τ from Eqs. (b) and (c) gives

$$\frac{du}{dy} = \frac{1}{\mu} \frac{dp}{dx} y \quad (d)$$

Treating dp/dx as a constant and integrating with respect to y gives

$$u = \frac{1}{2\mu} \frac{dp}{dx} y^2 + C_1 \quad (e)$$

At the boundaries, where $y = \pm c/2$, the velocity u is zero. Using one of these conditions in Eq. (e) gives

$$0 = \frac{1}{2\mu} \frac{dp}{dx} \left(\frac{c}{2} \right)^2 + C_1$$

or

$$C_1 = -\frac{c^2}{8\mu} \frac{dp}{dx}$$

Substituting C_1 in Eq. (e) yields

$$u = \frac{1}{8\mu} \frac{dp}{dx} (4y^2 - c^2) \quad (f)$$

Assuming the pressure varies linearly from p_s to 0 at $x = 0$ to l' , respectively, the pressure can be written as

$$p = p_s - \frac{p_s}{l'} x \quad (g)$$

and therefore the pressure gradient is given by

$$\frac{dp}{dx} = -\frac{p_s}{l'} \quad (h)$$

We can now substitute Eq. (h) in Eq. (f) and the relationship between the oil velocity and the coordinate y is

$$u = \frac{p_s}{8\mu l'} (c^2 - 4y^2) \quad (12-21)$$

Figure 12-29 shows a graph of this relation fitted into the clearance space c so that you can see how the velocity of the lubricant varies from the journal surface to the bearing surface. The distribution is parabolic, as shown, with the maximum velocity occurring at the center, where $y = 0$. The magnitude is, from Eq. (12-21),

$$u_{\max} = \frac{p_s c^2}{8\mu l'} \quad (i)$$

To consider eccentricity, as shown in Fig. 12-30, the film thickness is $h = c - e \cos \theta$. Substituting h for c in Eq. (i), with the average ordinate of a parabola being two-thirds the maximum, the average velocity at any angular position θ is

$$u_{\text{av}} = \frac{2}{3} \frac{p_s h^2}{8\mu l'} = \frac{p_s}{12\mu l'} (c - e \cos \theta)^2 \quad (j)$$

We still have a little further to go in this analysis, so please be patient. Now that we have an expression for the lubricant velocity, we can compute the amount of lubricant

Figure 12-29

Parabolic distribution of the lubricant velocity.

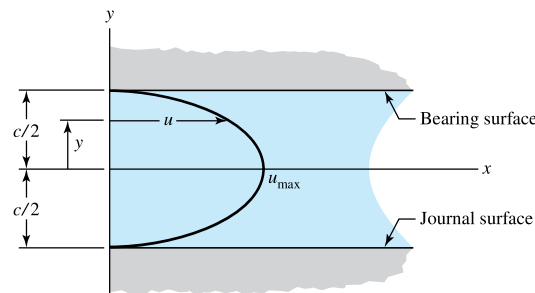
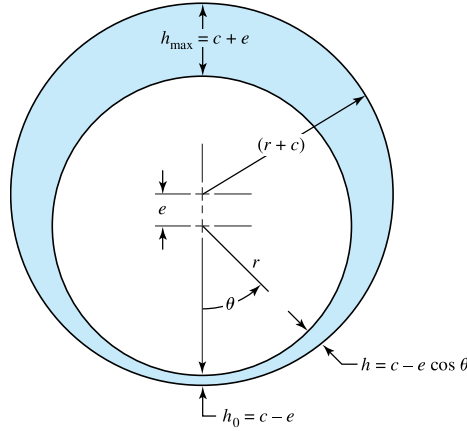


Figure 12-30



that flows out both ends. The elemental side flow at any position θ (Fig. 12-30) is

$$dQ_s = 2u_{av}dA = 2u_{av}(rh d\theta) \quad (k)$$

where dA is the elemental area. Substituting u_{av} from Eq. (j) and (h) from Fig. 12-30 gives

$$dQ_s = \frac{p_s r}{6\mu l'} (c - e \cos \theta)^3 d\theta \quad (l)$$

Integrating around the bearing gives the total side flow as

$$Q_s = \int dQ_s = \frac{p_s r}{6\mu l'} \int_0^{2\pi} (c - e \cos \theta)^3 d\theta = \frac{p_s r}{6\mu l'} (2\pi c^3 + 3\pi c e^2)$$

Rearranging, with $\epsilon = e/c$, gives

$$Q_s = \frac{\pi p_s r c^3}{3\mu l'} (1 + 1.5\epsilon^2) \quad (12-22)$$

In analyzing the performance of pressure-fed bearings, the bearing length should be taken as l' , as defined in Fig. 12-28. The characteristic pressure in each of the two bearings that constitute the pressure-fed bearing assembly P is given by

$$P = \frac{W/2}{2rl'} = \frac{W}{4rl'} \quad (12-23)$$

The charts for flow variable and flow ratio (Figs. 12-19 and 12-20) do not apply to pressure-fed bearings. Also, the maximum film pressure given by Fig. 12-21 must be increased by the oil supply pressure p_s to obtain the total film pressure.

Since the oil flow has been increased by forced feed, Eq. (12-14) will give a temperature rise that is too high because the side flow carries away all the heat generated. The plumbing in a pressure-fed bearing is depicted schematically in Fig. 12-31. The oil leaves the sump at the externally maintained temperature T_s at the volumetric rate Q_s . The heat gain of the fluid passing through the bearing is

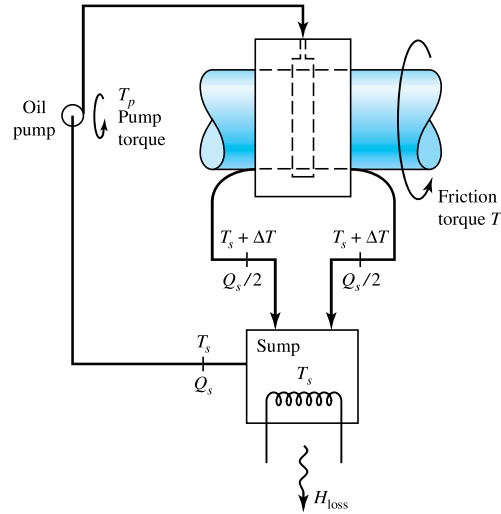
$$H_{\text{gain}} = 2\rho C_p (Q_s/2) \Delta T = \rho C_p Q_s \Delta T \quad (m)$$

At steady state, the rate at which the journal does frictional work on the fluid film is

$$H_f = \frac{2\pi TN}{J} = \frac{2\pi f WrN}{J} = \frac{2\pi W N c}{J} \frac{fr}{c} \quad (n)$$

Figure 12-31

Pressure-fed centrally located full annular-groove journal bearing with external, coiled lubricant sump.



Equating the heat gain to the frictional work and solving for ΔT gives

$$\Delta T = \frac{2\pi W N c}{J \rho C_p Q_s} \frac{f r}{c} \quad (o)$$

Substituting Eq. (12-22) for Q_s in the equation for ΔT gives

$$\Delta T = \frac{2\pi}{J \rho C_p} W N c \frac{f r}{c} \frac{3\mu l'}{(1 + 1.5\epsilon^2)\pi p_s r c^3}$$

The Sommerfeld number may be expressed as

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{r}{c}\right)^2 \frac{4rl'\mu N}{W}$$

Solving for $\mu N l'$ in the Sommerfeld expression; substituting in the ΔT expression; and using $J = 9336 \text{ lbf} \cdot \text{in/Btu}$, $\rho = 0.0311 \text{ lbf/in}^3$, and $C_p = 0.42 \text{ Btu/(lbf} \cdot \text{°F)}$, we find

$$\Delta T_F = \frac{3(fr/c)SW^2}{2J\rho C_p p_s r^4} \frac{1}{(1 + 1.5\epsilon^2)} = \frac{0.0123(fr/c)SW^2}{(1 + 1.5\epsilon^2)p_s r^4} \quad (12-24)$$

where ΔT_F is ΔT in $^{\circ}\text{F}$. The corresponding equation in SI units uses the bearing load W in kN, lubricant supply pressure p_s in kPa, and the journal radius r in mm:

$$\Delta T_C = \frac{978(10^6)}{1 + 1.5\epsilon^2} \frac{(fr/c)SW^2}{p_s r^4} \quad (12-25)$$

An analysis example of a pressure-fed bearing will be useful.

EXAMPLE 12-6

A circumferential-groove pressure-fed bearing is lubricated with SAE grade 20 oil supplied at a gauge pressure of 30 psi. The journal diameter d_j is 1.750 in, with a unilateral tolerance of -0.002 in. The central circumferential bushing has a diameter d_b of 1.753 in, with a unilateral tolerance of $+0.004$ in. The l'/d ratio of the two “half-bearings” that constitute the complete pressure-fed bearing is $1/2$. The journal

angular speed is 3000 rev/min, or 50 rev/s, and the radial steady load is 900 lbf. The external sump is maintained at 120°F as long as the necessary heat transfer does not exceed 800 Btu/h.

- (a) Find the steady-state average film temperature.
 (b) Compare h_0 , T_{\max} , and P_{st} with the Trumpler criteria.
 (c) Estimate the volumetric side flow Q_s , the heat loss rate H_{loss} , and the parasitic friction torque.

Solution

(a)

$$r = \frac{d_j}{2} = \frac{1.750}{2} = 0.875 \text{ in}$$

$$c_{\min} = \frac{(d_b)_{\min} - (d_j)_{\max}}{2} = \frac{1.753 - 1.750}{2} = 0.0015 \text{ in}$$

Since $l'/d = 1/2$, $l' = d/2 = r = 0.875 \text{ in}$. Then the pressure due to the load is

$$P = \frac{W}{4rl'} = \frac{900}{4(0.875)(0.875)} = 294 \text{ psi}$$

The Sommerfeld number S can be expressed as

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{0.875}{0.0015}\right)^2 \frac{\mu'}{(10^6)} \frac{50}{294} = 0.0579\mu' \quad (1)$$

We will use a tabulation method to find the average film temperature. The first trial average film temperature \bar{T}_f will be 170°F. Using the Seireg curve fit of Table 12–1, we obtain

$$\mu' = 0.0136 \exp[1271.6/(170 + 95)] = 1.650 \mu\text{reyn}$$

From Eq. (1)

$$S = 0.0579\mu' = 0.0579(1.650) = 0.0955$$

From Fig. (12–18), $fr/c = 3.3$, and from Fig. (12–16), $\epsilon = 0.80$. From Eq. (12–24),

$$\Delta T_F = \frac{0.0123(3.3)(0.0955)(900^2)}{[1 + 1.5(0.80)^2]30(0.875^4)} = 91.1^\circ\text{F}$$

$$T_{\text{av}} = T_s + \frac{\Delta T}{2} = 120 + \frac{91.1}{2} = 165.6^\circ\text{F}$$

We form a table, adding a second line with $\bar{T}_f = 168.5^\circ\text{F}$:

Trial	\bar{T}_f	μ'	S	fr/c	ϵ	ΔT_F	T_{av}
1	170	1.65	0.0955	3.3	0.800	91.1	165.6
2	168.5	1.693	0.0980	3.39	0.792	97.1	168.5

If the iteration had not closed, one could plot trial \bar{T}_f against resulting T_{av} and draw a straight line between them, the intersection with a $\bar{T}_f = T_{\text{av}}$ line defining the new trial \bar{T}_f .

Answer The result of this tabulation is $\bar{T}_f = 168.5^\circ\text{F}$, $\Delta T_F = 97.1^\circ\text{F}$, and $T_{\max} = 120 + 97.1 = 217.1^\circ\text{F}$

(b) Since $h_0 = (1 - \epsilon)c$,

$$h_0 = (1 - 0.792)0.0015 = 0.000312 \text{ in}$$

The required four Trumpler criteria, from “Significant Angular Speed” in Sec. 12–7 are

$$h_0 \geq 0.0002 + 0.00004(1.750) = 0.000270 \text{ in} \quad (\text{OK})$$

Answer $T_{\max} = T_s + \Delta T = 120 + 97.1 = 217.1^\circ\text{F} \quad (\leq 250^\circ\text{F. OK})$

$$P_{st} = \frac{W_{st}}{4rl'} = \frac{900}{4(0.875)0.875} = 294 \text{ psi} \quad (\leq 300 \text{ psi. OK})$$

Since we are close to the limit on P_{st} , the factor of safety on the load is approximately unity. ($n_d < 2$. Not OK.)

(c) From Eq. (12–22),

Answer
$$Q_s = \frac{\pi(30)0.875(0.0015)^3}{3(1.693)10^{-6}(0.875)}[1 + 1.5(0.80)^2] = 0.123 \text{ in}^3/\text{s}$$

$$H_{\text{loss}} = \rho C_p Q_s \Delta T = 0.0311(0.42)0.123(97.1) = 0.156 \text{ Btu/s}$$

or 562 Btu/h or 0.221 hp. The parasitic friction torque T is

Answer
$$T = fWr = \frac{fr}{c}Wc = 3.39(900)0.0015 = 4.58 \text{ lbf} \cdot \text{in}$$

12–12 Loads and Materials

Some help in choosing unit loads and bearing materials is afforded by Tables 12–5 and 12–6. Since the diameter and length of a bearing depend upon the unit load, these tables will help the designer to establish a starting point in the design.

Table 12–5

Range of Unit Loads in
Current Use for Sleeve
Bearings

Application	Unit Load	
	psi	MPa
Diesel engines:		
Main bearings	900–1700	6–12
Crankpin	1150–2300	8–15
Wristpin	2000–2300	14–15
Electric motors	120–250	0.8–1.5
Steam turbines	120–250	0.8–1.5
Gear reducers	120–250	0.8–1.5
Automotive engines:		
Main bearings	600–750	4–5
Crankpin	1700–2300	10–15
Air compressors:		
Main bearings	140–280	1–2
Crankpin	280–500	2–4
Centrifugal pumps	100–180	0.6–1.2

The length-diameter ratio l/d of a bearing depends upon whether it is expected to run under thin-film-lubrication conditions. A long bearing (large l/d ratio) reduces the coefficient of friction and the side flow of oil and therefore is desirable where thin-film or boundary-value lubrication is present. On the other hand, where forced-feed or positive lubrication is present, the l/d ratio should be relatively small. The short bearing length results in a greater flow of oil out of the ends, thus keeping the bearing cooler. Current practice is to use an l/d ratio of about unity, in general, and then to increase this ratio if thin-film lubrication is likely to occur and to decrease it for thick-film lubrication or high temperatures. If shaft deflection is likely to be severe, a short bearing should be used to prevent metal-to-metal contact at the ends of the bearings.

You should always consider the use of a partial bearing if high temperatures are a problem, because relieving the non-load-bearing area of a bearing can very substantially reduce the heat generated.

The two conflicting requirements of a good bearing material are that it must have a satisfactory compressive and fatigue strength to resist the externally applied loads and that it must be soft and have a low melting point and a low modulus of elasticity. The second set of requirements is necessary to permit the material to wear or break in, since the material can then conform to slight irregularities and absorb and release foreign particles. The resistance to wear and the coefficient of friction are also important because all bearings must operate, at least for part of the time, with thin-film or boundary lubrication.

Additional considerations in the selection of a good bearing material are its ability to resist corrosion and, of course, the cost of producing the bearing. Some of the commonly used materials are listed in Table 12–6, together with their composition and characteristics.

Bearing life can be increased very substantially by depositing a layer of babbitt, or other white metal, in thicknesses from 0.001 to 0.014 in over steel backup material. In fact, a copper-lead layer on steel to provide strength, combined with a babbitt overlay to enhance surface conformability and corrosion resistance, makes an excellent bearing.

Small bushings and thrust collars are often expected to run with thin-film or boundary lubrication. When this is the case, improvements over a solid bearing material can

Table 12–6

Some Characteristics
of Bearing Alloys

Alloy Name	Thickness, in	SAE Number	Clearance Ratio r/c	Load Capacity	Corrosion Resistance
Tin-base babbitt	0.022	12	600–1000	1.0	Excellent
Lead-base babbitt	0.022	15	600–1000	1.2	Very good
Tin-base babbitt	0.004	12	600–1000	1.5	Excellent
Lead-base babbitt	0.004	15	600–1000	1.5	Very good
Leaded bronze	Solid	792	500–1000	3.3	Very good
Copper-lead	0.022	480	500–1000	1.9	Good
Aluminum alloy	Solid		400–500	3.0	Excellent
Silver plus overlay	0.013	17P	600–1000	4.1	Excellent
Cadmium (1.5% Ni)	0.022	18	400–500	1.3	Good
Trimetal 88*				4.1	Excellent
Trimetal 77†				4.1	Very good

*This is a 0.008-in layer of copper-lead on a steel back plus 0.001 in of tin-base babbitt.

†This is a 0.013-in layer of copper-lead on a steel back plus 0.001 in of lead-base babbitt.

be made to add significantly to the life. A powder-metallurgy bushing is porous and permits the oil to penetrate into the bushing material. Sometimes such a bushing may be enclosed by oil-soaked material to provide additional storage space. Bearings are frequently ball-indented to provide small basins for the storage of lubricant while the journal is at rest. This supplies some lubrication during starting. Another method of reducing friction is to indent the bearing wall and to fill the indentations with graphite.

With all these tentative decisions made, a lubricant can be selected and the hydrodynamic analysis made as already presented. The values of the various performance parameters, if plotted as in Fig. 12–25, for example, will then indicate whether a satisfactory design has been achieved or additional iterations are necessary.

12–13 Bearing Types

A bearing may be as simple as a hole machined into a cast-iron machine member. It may still be simple yet require detailed design procedures, as, for example, the two-piece grooved pressure-fed connecting-rod bearing in an automotive engine. Or it may be as elaborate as the large water-cooled, ring-oiled bearings with built-in reservoirs used on heavy machinery.

Figure 12–32 shows two types of bushings. The solid bushing is made by casting, by drawing and machining, or by using a powder-metallurgy process. The lined bushing is usually a split type. In one method of manufacture the molten lining material is cast continuously on thin strip steel. The babitted strip is then processed through presses, shavers, and broaches, resulting in a lined bushing. Any type of grooving may be cut into the bushings. Bushings are assembled as a press fit and finished by boring, reaming, or burnishing.

Flanged and straight two-piece bearings are shown in Fig. 12–33. These are available in many sizes in both thick- and thin-wall types, with or without lining material. A locking lug positions the bearing and effectively prevents axial or rotational movement of the bearing in the housing.

Some typical groove patterns are shown in Fig. 12–34. In general, the lubricant may be brought in from the end of the bushing, through the shaft, or through the bushing. The flow may be intermittent or continuous. The preferred practice is to bring

Figure 12–32

Sleeve bushings.

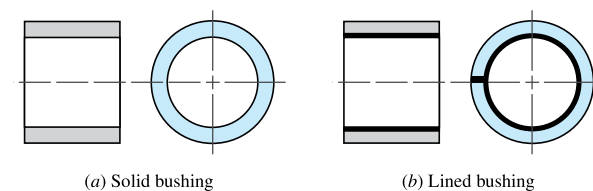


Figure 12–33

Two-piece bushings.

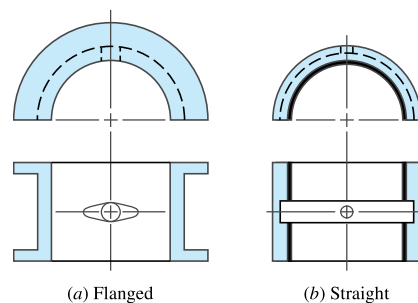
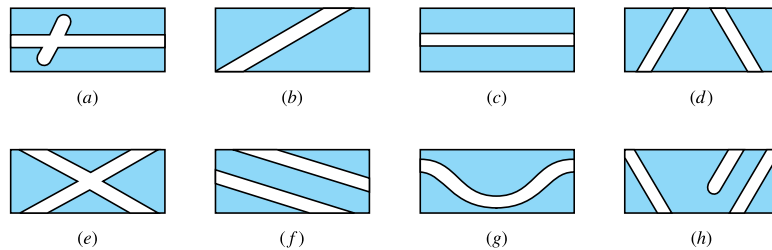


Figure 12-34

Developed views of typical groove patterns. (Courtesy of the Cleveland Graphite Bronze Company, Division of Clevite Corporation.)



the oil in at the center of the bushing so that it will flow out both ends, thus increasing the flow and cooling action.

12-14 Thrust Bearings

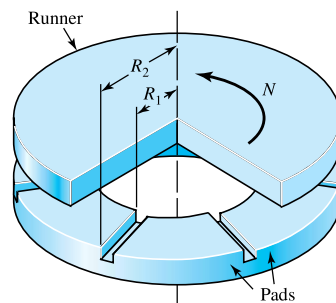
This chapter is devoted to the study of the mechanics of lubrication and its application to the design and analysis of journal bearings. The design and analysis of thrust bearings is an important application of lubrication theory, too. A detailed study of thrust bearings is not included here, because it would not contribute anything significantly different and because of space limitations. Having studied this chapter, you should experience no difficulty in reading the literature on thrust bearings and applying that knowledge to actual design situations.¹¹

Figure 12-35 shows a fixed-pad thrust bearing consisting essentially of a runner sliding over a fixed pad. The lubricant is brought into the radial grooves and pumped into the wedge-shaped space by the motion of the runner. Full-film, or hydrodynamic, lubrication is obtained if the speed of the runner is continuous and sufficiently high, if the lubricant has the correct viscosity, and if it is supplied in sufficient quantity. Figure 12-36 provides a picture of the pressure distribution under conditions of full-film lubrication.

We should note that bearings are frequently made with a flange, as shown in Fig. 12-37. The flange positions the bearing in the housing and also takes a thrust load. Even when it is grooved, however, and has adequate lubrication, such an arrangement is not theoretically a hydrodynamically lubricated thrust bearing. The reason for this is that the clearance space is not wedge-shaped but has a uniform thickness. Similar reasoning would apply to various designs of thrust washers.

Figure 12-35

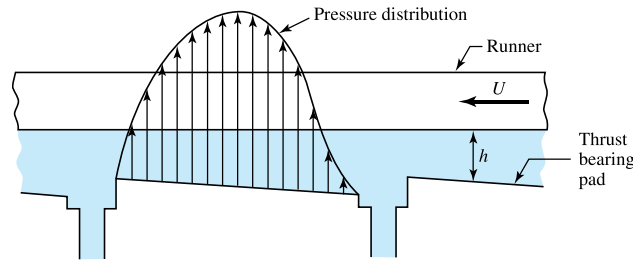
Fixed-pad thrust bearing. (Courtesy of Westinghouse Electric Corporation.)



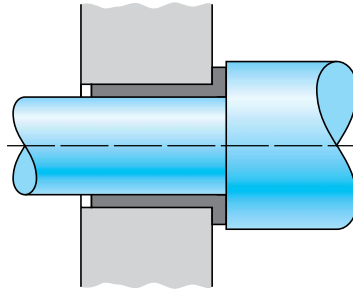
¹¹Harry C. Rippel, *Cast Bronze Thrust Bearing Design Manual*, International Copper Research Association, Inc., 825 Third Ave., New York, NY 10022, 1967. CBBI, 14600 Detroit Ave., Cleveland, OH, 44107, 1967.

Figure 12-36

Pressure distribution of lubricant in a thrust bearing.
(Courtesy of Copper Research Corporation.)

**Figure 12-37**

Flanged sleeve bearing takes both radial and thrust loads.



12-15 Boundary-Lubricated Bearings

When two surfaces slide relative to each other with only a partial lubricant film between them, *boundary lubrication* is said to exist. Boundary- or thin-film lubrication occurs in hydrodynamically lubricated bearings when they are starting or stopping, when the load increases, when the supply of lubricant decreases, or whenever other operating changes happen to occur. There are, of course, a very large number of cases in design in which boundary-lubricated bearings must be used because of the type of application or the competitive situation.

The coefficient of friction for boundary-lubricated surfaces may be greatly decreased by the use of animal or vegetable oils mixed with the mineral oil or grease. Fatty acids, such as stearic acid, palmitic acid, or oleic acid, or several of these, which occur in animal and vegetable fats, are called *oiliness agents*. These acids appear to reduce friction, either because of their strong affinity for certain metallic surfaces or because they form a soap film that binds itself to the metallic surfaces by a chemical reaction. Thus the fatty-acid molecules bind themselves to the journal and bearing surfaces with such great strength that the metallic asperities of the rubbing metals do not weld or shear.

Fatty acids will break down at temperatures of 250°F or more, causing increased friction and wear in thin-film-lubricated bearings. In such cases the *extreme-pressure*, or EP, lubricants may be mixed with the fatty-acid lubricant. These are composed of chemicals such as chlorinated esters or tricresyl phosphate, which form an organic film between the rubbing surfaces. Though the EP lubricants make it possible to operate at higher temperatures, there is the added possibility of excessive chemical corrosion of the sliding surfaces.

When a bearing operates partly under hydrodynamic conditions and partly under dry or thin-film conditions, a *mixed-film lubrication* exists. If the lubricant is supplied by hand oiling, by drop or mechanical feed, or by wick feed, for example, the bearing

is operating under mixed-film conditions. In addition to occurring with a scarcity of lubricant, mixed-film conditions may be present when

- The viscosity is too low.
- The bearing speed is too low.
- The bearing is overloaded.
- The clearance is too tight.
- Journal and bearing are not properly aligned.

Relative motion between surfaces in contact in the presence of a lubricant is called *boundary lubrication*. This condition is present in hydrodynamic film bearings during starting, stopping, overloading, or lubricant deficiency. Some bearings are boundary lubricated (or dry) at all times. To signal this an adjective is placed before the word “bearing.” Commonly applied adjectives (to name a few) are thin-film, boundary friction, Oilite, Oiles, and bushed-pin. The applications include situations in which thick film will not develop and there are low journal speed, oscillating journal, padded slides, light loads, and lifetime lubrication. The characteristics include considerable friction, ability to tolerate expected wear without losing function, and light loading. Such bearings are limited by lubricant temperature, speed, pressure, galling, and cumulative wear. Table 12–7 gives some properties of a range of bushing materials.

Linear Sliding Wear

Consider the sliding block depicted in Fig. 12–38, moving along a plate with contact pressure P' acting over area A , in the presence of a coefficient of sliding friction f_s . The linear measure of wear w is expressed in inches or millimeters. The work done by force $f_s PA$ during displacement S is $f_s PAS$ or $f_s PAVt$, where V is the sliding velocity and t is time. The material volume removed due to wear is wA and is proportional to the work done, that is, $wA \propto f_s PAVt$, or

$$wA = KPAVt$$

Table 12–7

Some Materials for
Boundary-Lubricated
Bearings and Their
Operating Limits

Material	Maximum Load, psi	Maximum Temperature, °F	Maximum Speed, fpm	Maximum PV Value*
Cast bronze	4 500	325	1 500	50 000
Porous bronze	4 500	150	1 500	50 000
Porous iron	8 000	150	800	50 000
Phenolics	6 000	200	2 500	15 000
Nylon	1 000	200	1 000	3 000
Teflon	500	500	100	1 000
Reinforced Teflon	2 500	500	1 000	10 000
Teflon fabric	60 000	500	50	25 000
Delrin	1 000	180	1 000	3 000
Carbon-graphite	600	750	2 500	15 000
Rubber	50	150	4 000	
Wood	2 000	150	2 000	15 000

* P = load, psi; V = speed, fpm.

Figure 12-38

Sliding block subjected to wear.

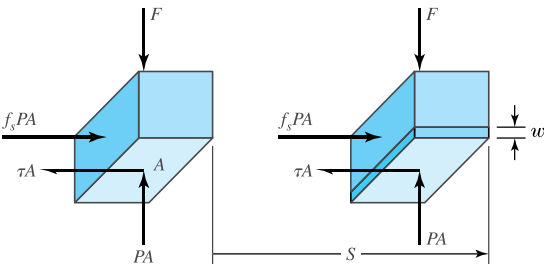


Table 12-8

Wear Factors in U.S. Customary Units*

Source: Oiles America Corp., Plymouth, MI 48170.

Bushing Material	Wear Factor K	Limiting PV
Oiles 800	$3(10^{-10})$	18 000
Oiles 500	$0.6(10^{-10})$	46 700
Polyactal copolymer	$50(10^{-10})$	5 000
Polyactal homopolymer	$60(10^{-10})$	3 000
66 nylon	$200(10^{-10})$	2 000
66 nylon + 15% PTFE	$13(10^{-10})$	7 000
+ 15% PTFE + 30% glass	$16(10^{-10})$	10 000
+ 2.5% MoS ₂	$200(10^{-10})$	2 000
6 nylon	$200(10^{-10})$	2 000
Polycarbonate + 15% PTFE	$75(10^{-10})$	7 000
Sintered bronze	$102(10^{-10})$	8 500
Phenol + 25% glass fiber	$8(10^{-10})$	11 500

*dim[K] = in³ · min/(lbf · ft · h), dim [PV] = psi · ft/min.

Table 12-9

Coefficients of Friction

Source: Oiles America Corp., Plymouth, MI 48170.

Type	Bearing	f_s
Placetic	Oiles 80	0.05
	Drymet ST	0.03
	Toughmet	0.05
Met	Cermet M	0.05
	Oiles 2000	0.03
	Oiles 300	0.03
	Oiles 500SP	0.03

where K is the proportionality factor, which includes f_s , and is determined from laboratory testing. The linear wear is then expressed as

$$w = KPVt \tag{12-26}$$

In US customary units, P is expressed in psi, V in fpm (i.e., ft/min), and t in hours. This makes the units of K in³ · min/(lbf · ft · h). SI units commonly used for K are cm³ · min/(kgf · m · h), where 1 kgf = 9.806 N. Tables 12-8 and 12-9 give some wear factors and coefficients of friction from one manufacturer.

Table 12-10

 Motion-Related Factor f_1

Mode of Motion	Characteristic Pressure P , psi	Velocity V , ft/min	f_1 *
Rotary	720 or less	3.3 or less	1.0
		3.3–33	1.0–1.3
		33–100	1.3–1.8
	720–3600	3.3 or less	1.5
		3.3–33	1.5–2.0
		33–100	2.0–2.7
Oscillatory	720 or less	>30°	3.3 or less
		3.3–100	1.3
		3.3–100	1.3–2.4
	720–3600	<30°	3.3 or less
		3.3–100	2.0
		3.3–100	2.0–3.6
Reciprocating	720 or less	>30°	3.3 or less
		3.3–100	2.0
		3.3–100	2.0–3.2
	720–3600	<30°	3.3 or less
		3.3–100	3.0
		3.3–100	3.0–4.8
	720 or less	33 or less	1.5
		33–100	1.5–3.8
		33 or less	2.0
	720–3600	33 or less	2.0
		33–100	2.0–7.5

 *Values of f_1 based on results over an extended period of time on automotive manufacturing machinery.

Table 12-11

 Environmental Factor f_2

 Source: Oiles America Corp.,
Plymouth, MI 48170.

Ambient Temperature, °F	Foreign Matter	f_2
140 or lower	No	1.0
140 or lower	Yes	3.0–6.0
140–210	No	3.0–6.0
140–210	Yes	6.0–12.0

It is useful to include a modifying factor f_1 depending on motion type, load, and speed and an environment factor f_2 to account for temperature and cleanliness conditions (see Tables 12-10 and 12-11). These factors account for departures from the laboratory conditions under which K was measured. Equation (12-26) can now be written as

$$w = f_1 f_2 K P V t \quad (12-27)$$

Wear, then, is proportional to PV , material property K , operating conditions f_1 and f_2 , and time t .

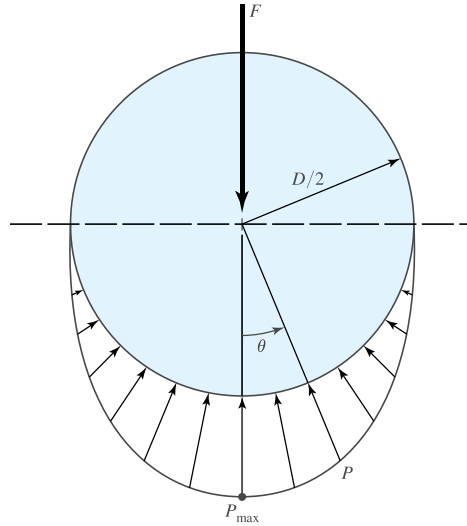
Bushing Wear

Consider a pin of diameter D , rotating at speed N , in a bushing of length L , and supporting a stationary radial load F . The nominal pressure P is given by

$$P = \frac{F}{DL} \quad (12-28)$$

Figure 12-39

Pressure distribution on a
boundary-lubricated bushing.



and if N is in rev/min and D is in inches, velocity in ft/min is given by

$$V = \frac{\pi DN}{12} \quad (12-29)$$

Thus PV , in psi · ft/min, is

$$PV = \frac{F}{DL} \frac{\pi DN}{12} = \frac{\pi}{12} \frac{FN}{L} \quad (12-30)$$

Note the independence of PV from the journal diameter D .

A time-wear equation similar to Eq. (12-27) can be written. However, before doing so, it is important to note that Eq. (12-28) provides the nominal value of P . Figure 12-39 provides a more accurate representation of the pressure distribution, which can be written as

$$p = P_{\max} \cos \theta \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

The vertical component of $p \, dA$ is $p \, dA \cos \theta = [pL(D/2) \, d\theta] \cos \theta = P_{\max}(DL/2) \cos^2 \theta \, d\theta$. Integrating this from $\theta = -\pi/2$ to $\pi/2$ yields F . Thus,

$$\int_{-\pi/2}^{\pi/2} P_{\max} \left(\frac{DL}{2} \right) \cos^2 \theta \, d\theta = \frac{\pi}{4} P_{\max} DL = F$$

or

$$P_{\max} = \frac{4}{\pi} \frac{F}{DL} \quad (12-31)$$

Substituting V from Eq. (12-29) and P_{\max} for P from Eq. (12-31) into Eq. (12-27) gives

$$w = f_1 f_2 K \frac{4}{\pi} \frac{F}{DL} \frac{\pi DNt}{12} = \frac{f_1 f_2 KFNt}{3L} \quad (12-32)$$

In designing a bushing, because of various trade-offs it is recommended that the length/diameter ratio be in the range

$$0.5 \leq L/D \leq 2 \quad (12-33)$$

EXAMPLE 12-7

An Oiles SP 500 alloy brass bushing is 1 in long with a 1-in bore and operates in a clean environment at 70°F. The allowable wear without loss of function is 0.005 in. The radial load is 700 lbf. The peripheral velocity is 33 ft/min. Estimate the number of revolutions for radial wear to be 0.005 in. See Fig. 12-40 and Table 12-12 from the manufacturer.

Solution

From Table 12-8, $K = 0.6(10^{-10}) \text{ in}^3 \cdot \text{min}/(\text{lbf} \cdot \text{ft} \cdot \text{h})$; Tables 12-10 and 12-11, $f_1 = 1.3$, $f_2 = 1$; and Table 12-12, $PV = 46\,700 \text{ psi} \cdot \text{ft}/\text{min}$, $P_{\max} = 3560 \text{ psi}$, $V_{\max} = 100 \text{ ft}/\text{min}$. From Eqs. (12-31), (12-29), and (12-30),

$$P_{\max} = \frac{4}{\pi} \frac{F}{DL} = \frac{4}{\pi} \frac{700}{(1)(1)} = 891 \text{ psi} < 3560 \text{ psi} \quad (\text{OK})$$

$$P = \frac{F}{DL} = \frac{700}{(1)(1)} = 700 \text{ psi}$$

$$V = 33 \text{ ft}/\text{min} < 100 \text{ ft}/\text{min} \quad (\text{OK})$$

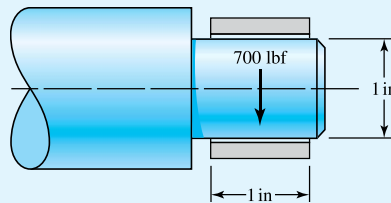
$$PV = 700(33) = 23\,100 \text{ psi} \cdot \text{ft}/\text{min} < 46\,700 \text{ psi} \cdot \text{ft}/\text{min} \quad (\text{OK})$$

Equation (12-32) with Eq. (12-29) is

$$w = f_1 f_2 K \frac{4}{\pi} \frac{F}{DL} \frac{\pi DN_t}{12} = f_1 f_2 K \frac{4}{\pi} \frac{F}{DL} V t$$

Figure 12-40

Journal/bushing for Ex. 12-7.

**Table 12-12**

Oiles 500 SP (SPBN · SPWN) Service Range and Properties

Source: Oiles America Corp., Plymouth, MI 48170.

Service Range	Units	Allowable
Characteristic pressure P_{\max}	psi	<3560
Velocity V_{\max}	ft/min	<100
PV product	(psi)(ft/min)	<46 700
Temperature T	°F	<300
Properties	Test Method, Units	Value
Tensile strength	(ASTM E8) psi	>110 000
Elongation	(ASTM E8) %	>12
Compressive strength	(ASTM E9) psi	49 770
Brinell hardness	(ASTM E10) HB	>210
Coefficient of thermal expansion	(10^{-5})°C	>1.6
Specific gravity		8.2

Solving for t gives

$$t = \frac{\pi DLw}{4f_1 f_2 KVF} = \frac{\pi(1)(1)0.005}{4(1.3)(1)0.6(10^{-10})33(700)} = 2180 \text{ h} = 130\,770 \text{ min}$$

The rotational speed is

$$N = \frac{12V}{\pi D} = \frac{12(33)}{\pi(1)} = 126 \text{ r/min}$$

Answer

$$\text{Cycles} = Nt = 126(130\,770) = 16.5(10^6) \text{ rev}$$

Temperature Rise

At steady state, the rate at which work is done against bearing friction equals the rate at which heat is transferred from the bearing housing to the surroundings by convection and radiation. The rate of heat generation in Btu/h is given by $f_s FV/J$, or

$$H_{\text{gen}} = \frac{f_s F(\pi D)(60N)}{12J} = \frac{5\pi f_s FDN}{J} \quad (12-34)$$

where N is journal speed in rev/min and $J = 778 \text{ ft} \cdot \text{lb}/\text{Btu}$. The rate at which heat is transferred to the surroundings, in Btu/h, is

$$H_{\text{loss}} = \bar{h}_{\text{CR}} A \Delta T = \bar{h}_{\text{CR}} A (T_b - T_\infty) = \frac{\bar{h}_{\text{CR}} A}{2} (T_f - T_\infty) \quad (12-35)$$

where A = housing surface area, ft^2

\bar{h}_{CR} = overall combined coefficient of heat transfer, $\text{Btu}/(\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F})$

T_b = housing metal temperature, $^\circ\text{F}$

T_f = lubricant temperature, $^\circ\text{F}$

The empirical observation that T_b is about midway between T_f and T_∞ has been incorporated in Eq. (12-35). Equating Eqs. (12-34) and (12-35) gives

$$T_f = T_\infty + \frac{10\pi f_s FDN}{J\bar{h}_{\text{CR}} A} \quad (12-36)$$

Although this equation seems to indicate the temperature rise $T_f - T_\infty$ is independent of length L , the housing surface area generally is a function of L . The housing surface area can be initially estimated, and as tuning of the design proceeds, improved results will converge. If the bushing is to be housed in a pillow block, the surface area can be roughly estimated from

$$A \approx \frac{2\pi DL}{144} \quad (12-37)$$

Substituting Eq. (12-37) into Eq. (12-36) gives

$$T_f \approx T_\infty + \frac{10\pi f_s FDN}{J\bar{h}_{\text{CR}}(2\pi DL/144)} = T_\infty + \frac{720 f_s FN}{J\bar{h}_{\text{CR}} L} \quad (12-38)$$

EXAMPLE 12-8

Choose an Oiles 500 bushing to give a maximum wear of 0.001 in for 800 h of use with a 300 rev/min journal and 50 lbf radial load. Use $\bar{h}_{CR} = 2.7 \text{ Btu}/(\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F})$, $T_{\max} = 300^\circ\text{F}$, $f_s = 0.03$, and a design factor $n_d = 2$. Table 12-13 lists the available bushing sizes from the manufacturer.

Solution

Using Eq. (12-38) with $n_d F$ for F , $f_s = 0.03$ from Table 12-9, and $\bar{h}_{CR} = 2.7 \text{ Btu}/(\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F})$, gives

$$L \geq \frac{720 f_s n_d F N}{J \bar{h}_{CR} (T_f - T_\infty)} = \frac{720(0.03)2(50)300}{778(2.7)(300 - 70)} = 1.34 \text{ in}$$

From Table 12-13, the smallest available bushing has an ID = $\frac{5}{8}$ in, OD = $\frac{7}{8}$ in, and $L = 1\frac{1}{2}$ in. However, for this case $L/D = 1.5/0.625 = 2.4$, and is outside of the recommendations of Eq. (12-33). Thus, for the first trial, try the bushing with ID = $\frac{3}{4}$ in, OD = $1\frac{1}{8}$ in, and $L = 1\frac{1}{2}$ in ($L/D = 1.5/0.75 = 2$). Thus,

Table 12-13

Available Bushing Sizes
(in inches) of One
Manufacturer*

ID	OD	L														
		$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	5	
$\frac{1}{2}$	$\frac{3}{4}$	•	•	•	•	•										
$\frac{5}{8}$	$\frac{7}{8}$		•	•		•		•								
$\frac{3}{4}$	$1\frac{1}{8}$		•	•		•		•								
$\frac{7}{8}$	$1\frac{1}{4}$			•		•	•	•								
1	$1\frac{3}{8}$			•		•	•	•	•	•						
1	$1\frac{1}{2}$			•		•		•		•						
$1\frac{1}{4}$	$1\frac{5}{8}$					•	•	•	•	•						
$1\frac{1}{2}$	2					•	•	•	•	•						
$1\frac{3}{4}$	$2\frac{1}{4}$						•	•	•	•	•	•	•	•		
2	$2\frac{1}{2}$							•		•	•	•				
$2\frac{1}{4}$	$2\frac{3}{4}$							•		•	•	•				
$2\frac{1}{2}$	3							•		•		•				
$2\frac{3}{4}$	$3\frac{3}{8}$							•		•	•	•				
3	$3\frac{5}{8}$									•	•	•	•			
$3\frac{1}{2}$	$4\frac{1}{8}$									•		•		•		
4	$4\frac{3}{4}$									•		•		•		
$4\frac{1}{2}$	$5\frac{3}{8}$											•		•	•	
5	6											•		•	•	

*In a display such as this a manufacturer is likely to show catalog numbers where the • appears.

$$\text{Eq. (12-31): } P_{\max} = \frac{4}{\pi} \frac{n_d F}{DL} = \frac{4}{\pi} \frac{2(50)}{0.75(1.5)} = 113 \text{ psi} < 3560 \text{ psi} \quad (\text{OK})$$

$$P = \frac{n_d F}{DL} = \frac{2(50)}{0.75(1.5)} = 88.9 \text{ psi}$$

$$\text{Eq. (12-29): } V = \frac{\pi DN}{12} = \frac{\pi(0.75)300}{12} = 58.9 \text{ ft/min} < 100 \text{ ft/min} \quad (\text{OK})$$

$$PV = 88.9(58.9) = 5240 \text{ psi} \cdot \text{ft/min} < 46\,700 \text{ psi} \cdot \text{ft/min} \quad (\text{OK})$$

From Table 12-10, interpolation gives

V	f_1
33	1.3
58.9	f_1 from which $f_1 = 1.50$
100	1.8

Eq. (12-32), with Tables 12-8 and 12-10:

$$w = \frac{f_1 f_2 K n_d F N t}{3L} = \frac{1.50(1)6(10^{-11})2(50)300(800)}{3(1.5)} = 0.000480 \text{ in} < 0.001 \text{ in} \quad (\text{OK})$$

Answer Select ID = $\frac{3}{4}$ in, OD = $1\frac{1}{8}$ in, and $L = 1\frac{1}{2}$ in.

PROBLEMS

- 12-1** A full journal bearing has a journal diameter of 25 mm, with a unilateral tolerance of -0.03 mm. The bushing bore has a diameter of 25.03 mm and a unilateral tolerance of 0.04 mm. The l/d ratio is $1/2$. The load is 1.2 kN and the journal runs at 1100 rev/min. If the average viscosity is $55 \text{ mPa} \cdot \text{s}$, find the minimum film thickness, the power loss, and the side flow for the minimum clearance assembly.
- 12-2** A full journal bearing has a journal diameter of 32 mm, with a unilateral tolerance of -0.012 mm. The bushing bore has a diameter of 32.05 mm and a unilateral tolerance of 0.032 mm. The bearing is 64 mm long. The journal load is 1.75 kN and it runs at a speed of 900 rev/min. Using an average viscosity of $55 \text{ mPa} \cdot \text{s}$ find the minimum film thickness, the maximum film pressure, and the total oil-flow rate for the minimum clearance assembly.
- 12-3** A journal bearing has a journal diameter of 3.000 in, with a unilateral tolerance of -0.001 in. The bushing bore has a diameter of 3.005 in and a unilateral tolerance of 0.004 in. The bushing is 1.5 in long. The journal speed is 600 rev/min and the load is 800 lbf. For both SAE 10 and SAE 40, lubricants, find the minimum film thickness and the maximum film pressure for an operating temperature of 150°F for the minimum clearance assembly.